MATH 340: Discrete Structures II. Winter 2017.
Due in class on Wednesday, March 8th.

Assignment #3: Discrete Probability.

1. *Bayes Theorem.*

a) *Babies.* A family has two children, and at least one of them is a boy. What is the probability that the family has one boy and one girl?

b) *A die and a coin.* Bob rolls a single six-sided die, and then flips a coin the number of times showing on the die. The coin comes up heads every time. What is the probability that the die showed 6?

c) *Drug test.* A large company gives a new employee a drug test. The False-Positive rate is 2% and the False-Negative rate is 10%. In addition, 1% of the population use the drug. The employee tests positive for the drug. What is the probability the employee uses the drug?

2. *Monty Hall variant.* Consider an alternative version of the problem where Monty is absent-minded and forgets where the prize is. You select a door. Monty opens one of the other two doors *at random.* If Montys door contains the prize you lose and the game is over. On the other hand, suppose Montys door did not contain the prize. Should you now switch doors when offered the chance?

3. *Linearity of expectation.* We play the following game. A fair die is rolled n times. Every time we roll a number that differs by one from the number we got on our previous roll we win \$1. Every time we roll a number that is equal to the one we got on our previous roll we lose \$1. (For example, if we get numbers $1, 3, 3, 4, 6, 5, 3$ as a result of seven rolls, then we win \$1 in total, getting \$1 on the fourth and sixth roll, and losing \$1 on the third roll.) What are our expected winnings?

4. *Markov inequality.* Let X be a random variable taking only non-negative values, and let c be a positive constant. Show that

$$p(X \geq c) \leq \mathrm{E}(X)/c.$$

5. *Binomial distribution.* The Stanley Cup winner is determined in the final series between two teams. The first team to win 4 games wins the Cup. Suppose that Montreal Canadiens advance to the final series, and they have a probability of 0.6 to win each game, and the game results are independent of each other. Find the probability that

a) Canadiens win the Stanley cup;

b) Seven games are required to determine the winner.

6. *Geometric distribution.* Suppose we run repeated independent Bernoulli trials (with success probability p) until we obtain a success. Let the random variable X be the number of trials needed before we obtain a success.

a) Calculate the probability $P(X = k)$ for an integer k.

b) Prove that $E(X) = 1/p$.