Abstract McGill-lecture (27-3-2001): Bob Coecke

From Birkhoff-von Neumann to Eilenberg-Mac Lane: Categories for the digesting physicist.

i) After a slightly iconoclastic overview of "practising theoretical physics", pointing at the areas where category theory comes in, we give an overview of the development of quantum logic, originating in the Birkhoff-von Neumann paper (1937), and both representational and operationally established by Piron (1964–1976). We discuss the different conceptions that exist of quantum logic, ranging from a metaphysical attack on classical reasoning, over a structural study of quantum theory, to a domain of mathematics that deals with non-... structures.

ii) Quoting Birkhoff-von Neumann (1937): "... whereas for logicians the orthocomplementation properties of negation were the ones least able to withstand a critical analysis, the study of mechanics points to the distributive identities as the weakest link in the algebra of logic." We present a new perspective on quantum logic that exactly does the opposite: we motivate that the Bruns-Lakser distributive hull construction (1970), embedding any meet-semilattice in a locale where the inclusion equips the latter with a distributive closure, provides an intuitionistic or disjunctive representation for quantum logic that also from an ontological perspective is definitely a more natural representation of physical (quantum) systems.

iii) In the past years there has been elaboration towards a dynamic quantum logic (Faure-Moore-Piron (1995): Deterministic Evolutions and Schrödinger Flows; *Helv. Phys. Acta* **68**, 150–157; Coecke-Moore-Stubbe (et al.): quant-ph/0008020; quant-ph/0008054; quant-ph/0009100; math.LO/0011208 & 0011209; Coecke (2001–IC): Do we have to retain Cartesianity in Topos-Approaches to Quantum Theory and Quantum Gravity?). We present the semantics of this approach where processes are encoded as morphisms of categories. It turns out that the above discussed disjunctive representation of quantum logic can be equipped with an "environment-labelled" additional structure of adjointly related monoidal and exponential operators that provide a dual linear structure respectively encoding propagation and causation for physical properties, i.e., a structural representation of the system's dynamic. On can indeed show that propagation of physical properties and assignment of causes of actuality (say truth) of physical properties are not inverse but adjoint. Moreover, this "causal duality" enables to derive the quantum theoretical representation of evolution and compoundness (i.e., tensor products) via Faure-Frölicher categorical projective geometry (1993–1995).