Restriction categories and categories of partial maps

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Abstract

Because partial maps are central to so many issues in computer science there has been a considerable effort to develop their theory: an early categorical attempt was that of Di Paola and Heller who introduced the notion of dominical categories. This was followed quickly by Robinson and Rosolini who simplified the categorical structure to P-categories, categories with a partial product structure. These were also considered by Bart Jacobs as the semantics of weakening and Carboni studied them from the bicategorical point of view.

These categories are also, essentially, what I called "copy categories" in a manuscript which started circulating in about 1995. It was never published for two main reasons. First, much of the material on partiality was already available in the above (and Mulry's) work. Secondly, one of the main motivating results, the description of the extensive completion of a distributive category, had also been proven independently by the Steve Lack at roughly the same time. Thus, it had been resolved that we should try to pool resources and publish the results jointly .... and that event had to wait for a time when we could get together physically!

We did get together in Sydney this last Australian winter and, inevitablely, we completely reworked our approach! Our new approach may be regarded as a return to the key ideas expressed in the opening sentence of Di Paola and Heller's paper on dominical categories where they emphasized the important role of idempotents in expressing partiality. We are now convinced of the value of directly legislating of the existence of idempotents to express partiality and we call the resulting algebraic structure a restriction category.

This structure has allowed us to develop surprisingly incisive basic results on categories of partial maps in an uncluttered manner. A slightly surprising results is that the category of restriction categories rCat_0 is monadic over Cat_0. Steve discovered this construction while I was trying to show him how formal subobjects (given by a fibration) could be universally adjoined to a category.

If there is time I would also like to describe our results on partial map classifiers and how these results were greatly facilitated by having the theory of restriction categories in hand.