Knights and Knaves

These problems are quoted (or sometimes modified from) the wonderful logic puzzle books of Raymond Smullyan, in particular his book *What is the name of this book?*

There is an island far off in the Pacific, called the island of Knights and Knaves. On this island, there are people called knights (who always tell the truth) and knaves (who always lie). They may be either male or female.

1. We have three people A, B, and C on the Island of Knights and Knaves. Suppose A and B say the following:

   A: All of us are knaves.

   B: Exactly one of us is a knave.

   Can it be determined what B is? Can it be determined what C is?

2. Suppose A says, "I am a knave but B isn't." What are A and B?

3. We again have three inhabitants, A, B and C, each of whom is a knight or a knave. Two people are said to be of the same type if they are both knights or both knaves. A and B make the following statements:

   A: B is a knave.

   B: A and C are of the same type.

   What is C?

4. Again three people A, B and C. A says "B and C are of the same type." Someone then asks C, "Are A and B of the same type?" What does C answer?

5. We have two people A, B, each of whom is either a knight or a knave. Suppose A makes the following statement: "If I am a knight, then so is B." Can it be determined what A and B are?

6. Someone asks A, "Are you a knight?" He replies, "If I'm a knight, then I'll eat my hat!" Prove that A has to eat his hat.

7. A says, "If I'm a knight, then two plus two equals four." Is A a knight or a knave?

8. A says, "If I'm a knight, then two plus two equals five." What would you conclude?

9. Given two people, A, B, both of whom are knights or knaves. A says, "If B is a knight then I am a knave." What are A and B?

10. Two individuals, X and Y, were being tried for participation in a robbery. A and B were court witnesses, and each of A, B is either a knight or a knave. The witnesses make the following statement:

    A: If X is guilty, so is Y.

    B: Either X is innocent or Y is guilty.

Are A and B necessarily of the same type? (i.e. either both knights or both knaves.)

11. On the island of knights and knives, three inhabitants A,B,C are being interviewed. A and B make the following statements:

    A: B is a knight.
    B: If A is a knight so is C.

    Can it be determined what any of A, B, C are?

12. Suppose the following two statements are true: (1) I love Betty or I love Jane. (2) If I love Betty then I love Jane. Does it necessarily follow that I love Betty? Does it necessarily follow that I love Jane?

13. Suppose someone asks me, "Is it really true that if you love Betty then you also love Jane?" I reply, "If it is true, then I love Betty." Does it follow that I love Betty? Does it follow that I love Jane?

14. This problem, though simple, is a bit surprising. Suppose it is given that I am either a knight or a knave. I make the following two statements:

    (a) I love Linda.
    (b) If I love Linda then I love Kathy.

    Am I a knight or a knave?

15. Is There Gold on This Island? On a certain island of knights and knaves, it is rumored that there is gold buried on the island. You arrive on the island and ask one of the natives, A, whether there is gold on this island. He makes the following response: "There is gold on this island if and only if I am a knight." Our problem has two parts:

    (a) Can it be determined whether A is a knight or a knave?

    (b) Can it be determined whether there is gold on the island?

16. Suppose, instead of A having volunteered this information, you had asked A, "Is the statement that you are a knight equivalent to the statement that there is gold on this island?" Had he answered "Yes," the problem would have reduced to the preceding one. Suppose he had answered "No." Could you then tell whether or not there is gold on the island?

17. The First Island. On the first Island he tried, he met two natives A, B, who made the following statements:

    A: B is a knight and this is the island of Maya.
    B: A is a knave and this is the island of Maya.

    Is this the island of Maya?

18. The Second Island. On this Island, two natives A, B, make the following statements:

    A: We are both knaves, and this is the island of Maya.
    B: That is true.

    Is this the island of Maya?

19. The Third Island. On this island, A and B said the following:

    A: At least one of us is a knave, and this is the island of Maya. B: That is true.

    Is this the island of Maya?

20. Here is a bit of an off-beat question. One day, on the island of Knights and Knaves, you see an inhabitant. You go up to her and ask: "Are you a knight or are you a knave ?" She says: "I won't tell you" and walks away. Is it possible to decide if she is a knight or a knave?