Lecture 16 (Sections 4.4, 4.7)

Coordinate systems

Theorem 7

Let $\mathcal{U} = \{\mathbf{u}_1, \dots, \mathbf{u}_n\}$ be a basis for H. Then the coefficients α_i in the expansion $\mathbf{x} = \alpha_1\mathbf{u}_1 + \dots + \alpha_n\mathbf{u}_n$ are uniquely determined by $\mathbf{x} \in H$.

Definition

The coefficients α_i in the expansion $\mathbf{x} = \alpha_1\mathbf{u}_1 + \dots + \alpha_n\mathbf{u}_n$ are called the coordinates of $\mathbf{x} \in H$ w.r.t. $\mathcal{U}$, and denoted by

$$[\mathbf{x}]_{\mathcal{U}} := \boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \dots \\ \alpha_n \end{bmatrix}$$

- When $H = \mathbb{R}^n$, we have $\mathbf{x} = P\boldsymbol{\alpha}$ with the change-of-coordinates matrix $P = [\mathbf{u}_1, \dots, \mathbf{u}_n]$
- $T : \mathbf{x} \mapsto \boldsymbol{\alpha} : H \longrightarrow \mathbb{R}^n$ is linear, one to one, and onto
- H is isomorphic to $\mathbb{R}^n$

Change of basis

Theorem 15

Let $\mathcal{U} = \{\mathbf{u}_1, \dots, \mathbf{u}_n\}$ and $\mathcal{V} = \{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ be bases for H. Then there is a unique $n \times n$ matrix P such that

$$[\mathbf{x}]_{\mathcal{V}} = P[\mathbf{x}]_{\mathcal{U}}$$

The columns of P are $[\mathbf{u}_1]_{\mathcal{V}}, \dots, [\mathbf{u}_n]_{\mathcal{V}}$, i.e., the vectors $\mathbf{u}_1, \dots, \mathbf{u}_n$ written in terms of the basis $\mathcal{V}$.