Lecture 11 (Sections 2.3 and 2.4)

LU factorization

LU factorization

$$A\mathbf{x} = \mathbf{b}$$

$$A = LU$$

- U is an echelon form of A
- L is unit lower triangular
- Solve $L\mathbf{y} = \mathbf{b}$ for $\mathbf{y}$ (forward substitution)
- then solve $U\mathbf{x} = \mathbf{y}$ for $\mathbf{x}$ (backward substitution)

$$\mathbf{b} = L\mathbf{y} = LU\mathbf{x} = A\mathbf{x}$$

LU factorization algorithm

Let A be $k \times n$ matrix, and L be $k \times k$

$$L = \begin{bmatrix} 1 & 0 & \dots & 0 \\ * & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ * & * & \dots & 1 \end{bmatrix}$$

- Reduce A to an echelon form U by a sequence of row replacement operations, if possible
- If the i-th row multiplied by t is added to the j-th row, place the number $-t$ in (j, i)-th position of L