Lecture 10 (Section 2.2)

Inverse of a matrix

Inverse of a matrix

- Let A be $n \times n$. $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one and onto $\Leftrightarrow$ A is invertible.
- $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible if and only if $ad - bc \neq 0$ and the inverse is

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- $\det A = ad - bc$ is called the determinant of A
- "invertible" = "nonsingular", "not invertible" = "singular"

Inverse of a matrix

Theorem 6

- A is invertible $\Rightarrow A^{-1}$ is invertible and

$$(A^{-1})^{-1} = A$$

- A and B have the same size and are invertible $\Rightarrow AB$ is invertible and

$$(AB)^{-1} = B^{-1}A^{-1}$$

- A is invertible $\Rightarrow A^T$ is invertible and

$$(A^T)^{-1} = (A^{-1})^T$$

Elementary matrices

- $n \times n$ matrix E is elementary if it is obtained from I_n by a single elementary row op.
- Every elementary matrix is invertible
- If A is $n \times k$ matrix,

$$\begin{aligned} I_n &\longrightarrow E \\ A &\longrightarrow EA \end{aligned}$$

- $n \times n$ matrix A is invertible $\Leftrightarrow A \sim I_n$

$$\begin{aligned} \Leftrightarrow \quad I_n &= E_p E_{p-1} \ldots E_1 A \\ \Leftrightarrow \quad A^{-1} &= E_p E_{p-1} \ldots E_1 = E_p E_{p-1} \ldots E_1 I_n \end{aligned}$$