Lecture 7 (Section 1.8)

General mappings

Linear mappings

General mappings

$$T : \mathbb{R}^n \to \mathbb{R}^k \qquad \text{or} \qquad \mathbf{x} \mapsto T(\mathbf{x})$$

- mapping (map, function, transformation) from $\mathbb{R}^n$ to $\mathbb{R}^k$
- T maps $\mathbf{x}$ to $T(\mathbf{x})$
- $\mathbb{R}^n$ is the domain of T
- $\mathbb{R}^k$ is the codomain of T
- for $\mathbf{x} \in \mathbb{R}^n$, $T(\mathbf{x}) \in \mathbb{R}^k$ is called the image of $\mathbf{x}$
- the range of T is the set of all $T(\mathbf{x})$

$$\text{Range}(T) = \{T(\mathbf{x}) : \mathbf{x} \in \mathbb{R}^n\}$$

Linear mappings

Definition

$T : \mathbb{R}^n \to \mathbb{R}^k$ is linear if

- $T(\alpha \mathbf{x}) = \alpha T(\mathbf{x})$ for all $\alpha \in \mathbb{R}$ and $\mathbf{x} \in \mathbb{R}^n$
- $T(\mathbf{x} + \mathbf{y}) = T(\mathbf{x}) + T(\mathbf{y})$ for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$

Theorem 9'

If $T : \mathbb{R}^n \to \mathbb{R}^k$ is a linear mapping, then

- $T(\mathbf{0}) = \mathbf{0}$
- $T(\alpha \mathbf{x} + \beta \mathbf{y}) = \alpha T(\mathbf{x}) + \beta T(\mathbf{y})$ for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and $\alpha, \beta \in \mathbb{R}$

Theorem 9"

If $T : \mathbb{R}^n \to \mathbb{R}^k$ satisfies

$$T(\alpha \mathbf{x} + \mathbf{y}) = \alpha T(\mathbf{x}) + T(\mathbf{y}) \qquad \text{for all } \mathbf{x}, \mathbf{y} \in \mathbb{R}^n \text{ and } \alpha \in \mathbb{R}$$

then T is linear.