The Binomial Distribution

The Binomial Distribution

Suppose random variable X can take values on the set

$$\{0, 1, \ldots, n\}$$

for some whole number $n \geq 1$. The **Binomial distribution** is a probability distribution for which

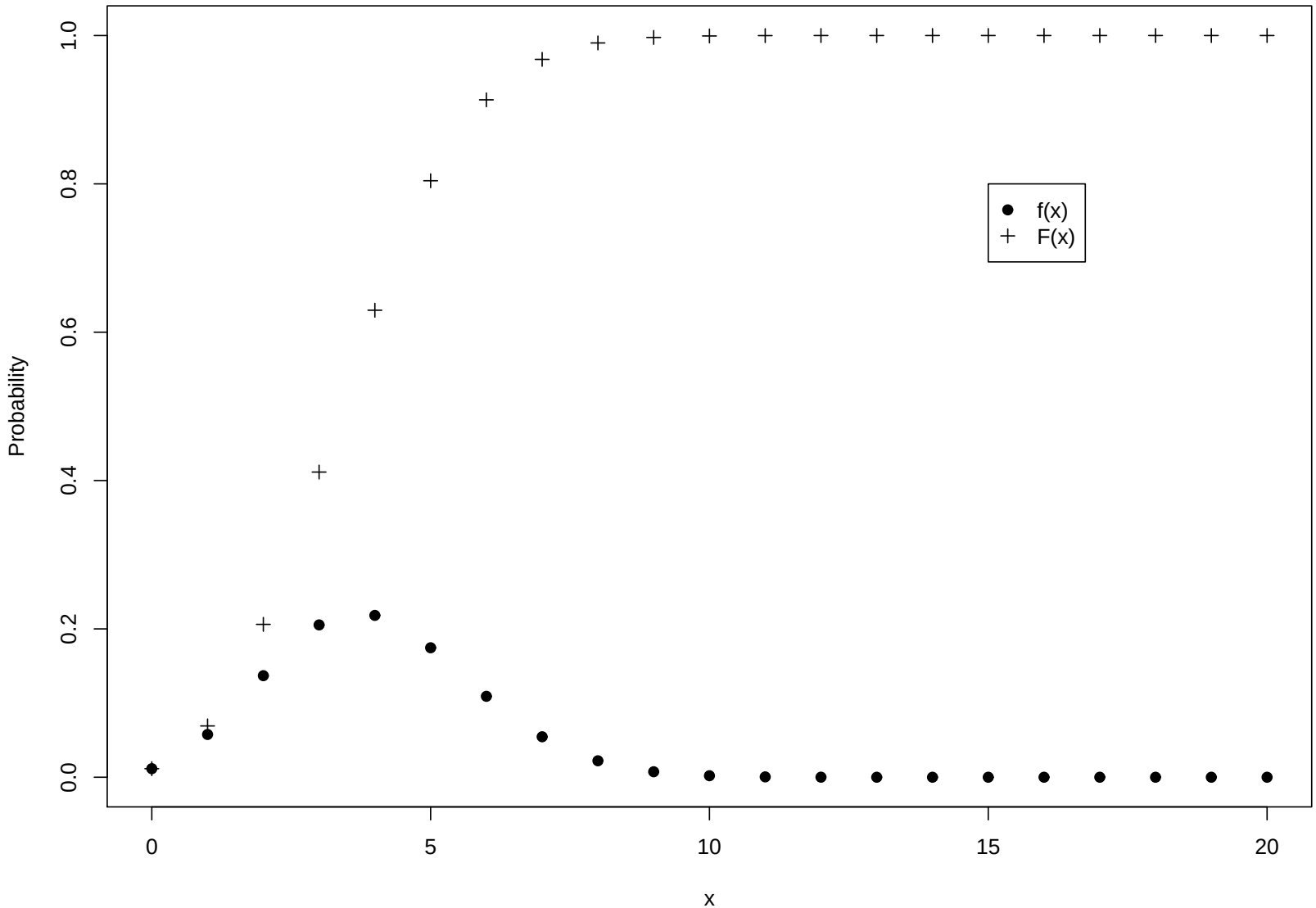
$$f(x) = \mathrm{P}[X = x] = \binom{n}{x} p^x (1-p)^{n-x} \qquad 0 \leq x \leq n$$

for some probability p.

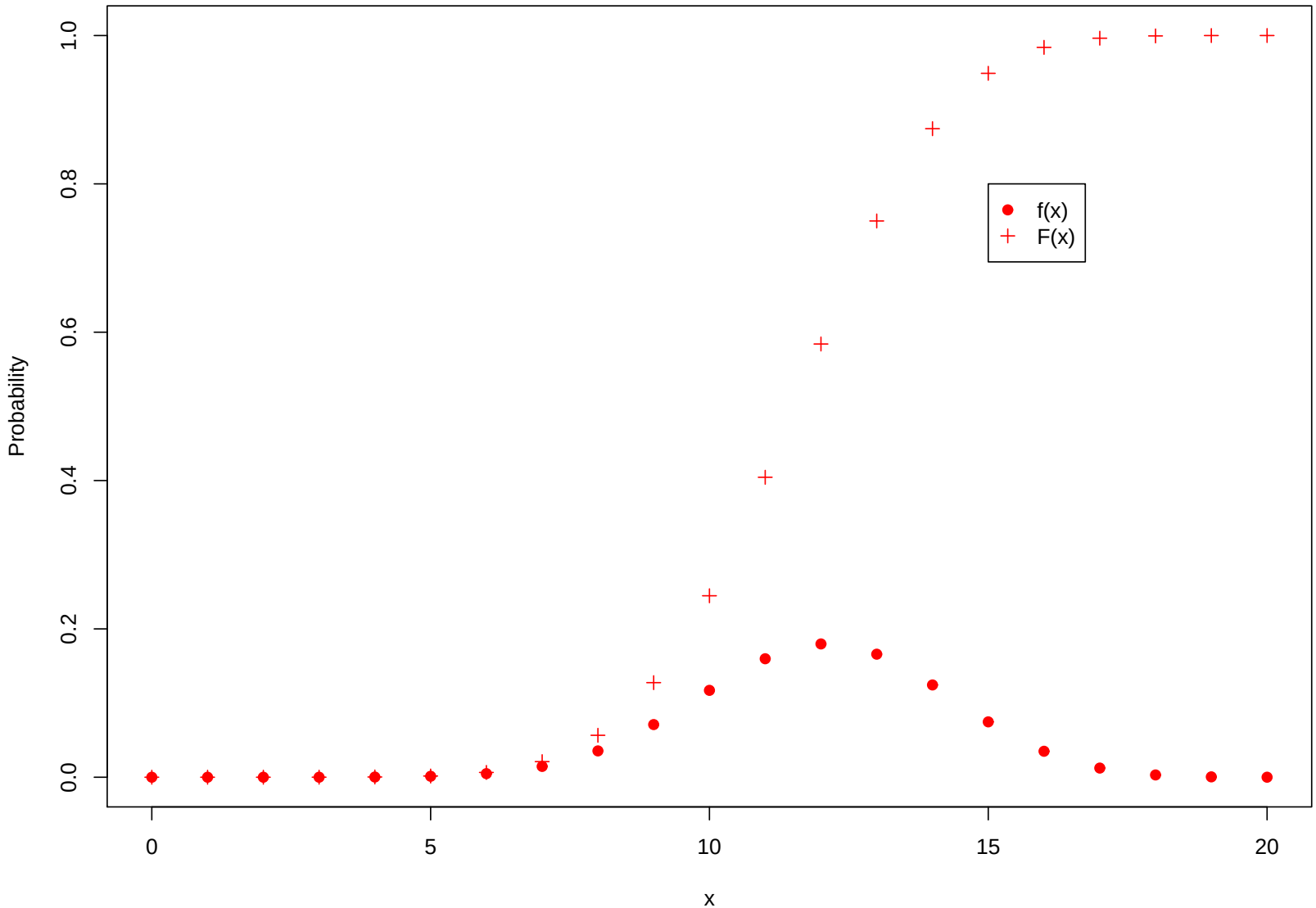
The **cumulative probability function**, $F(x)$, is defined by

$$F(x) = \mathrm{P}[X \leq x] = \sum_{t=0}^{x} \binom{n}{t} p^t (1-p)^{n-t}$$

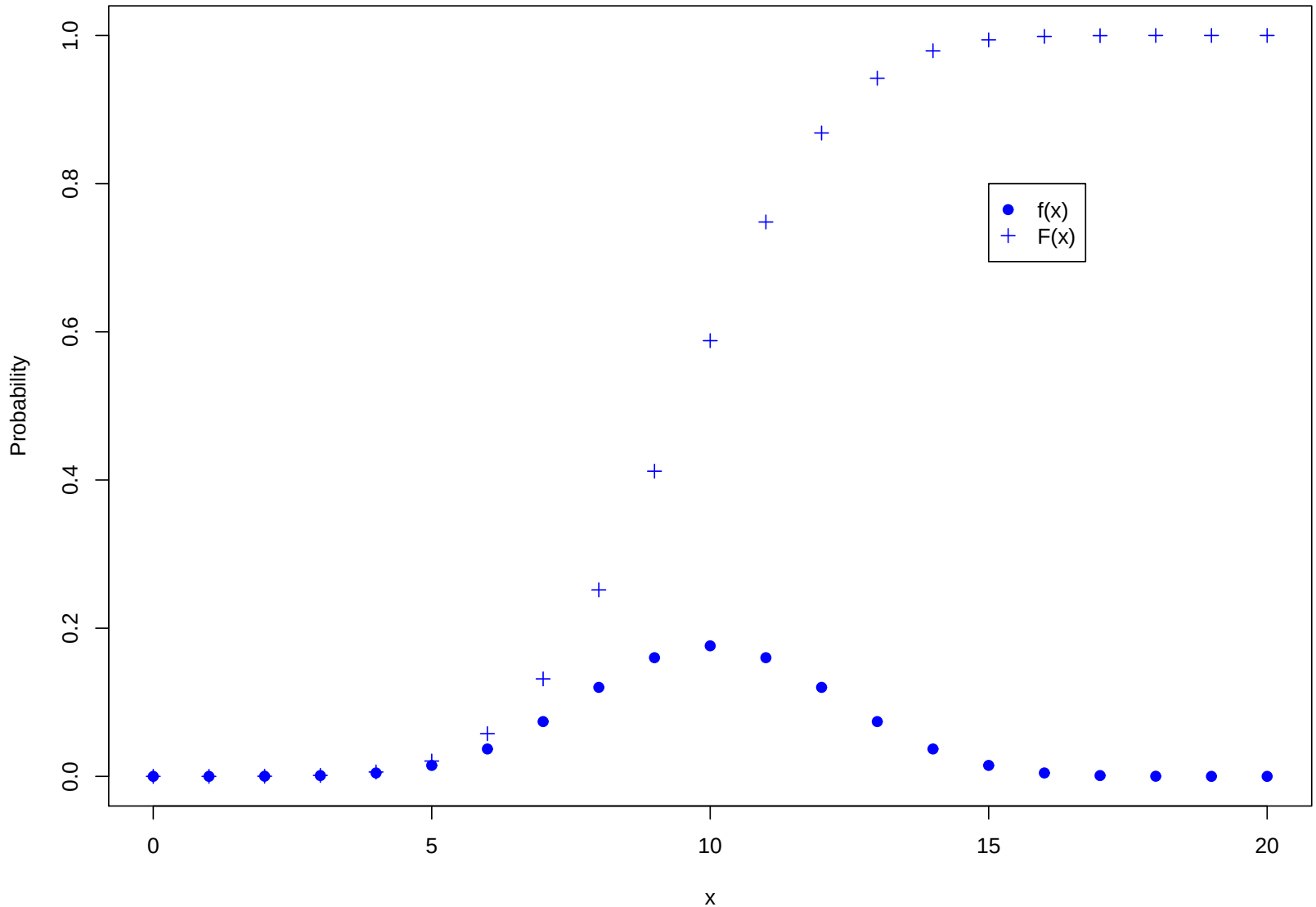
$n = 20, p = 0.2$



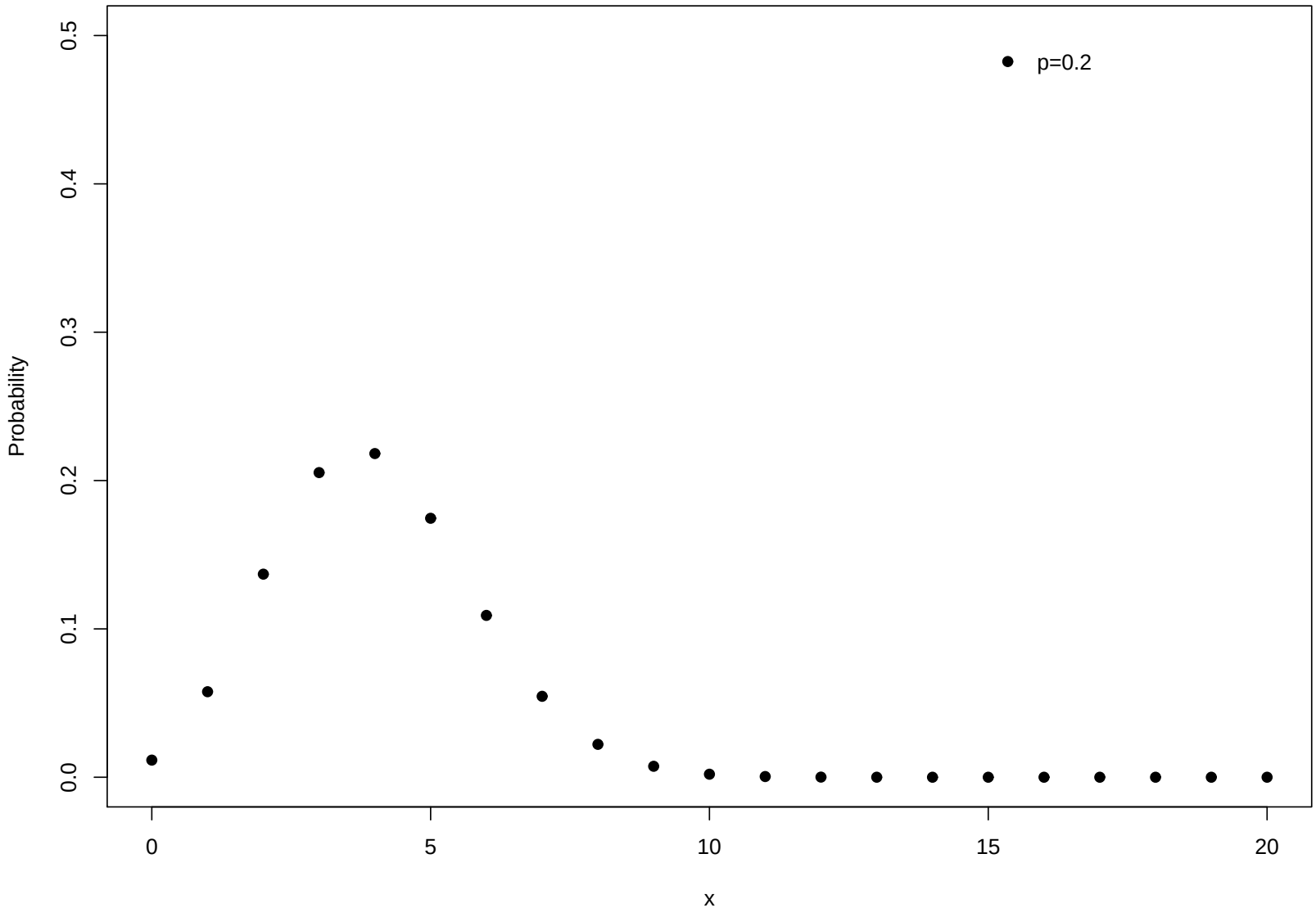
$n = 20, p = 0.6$



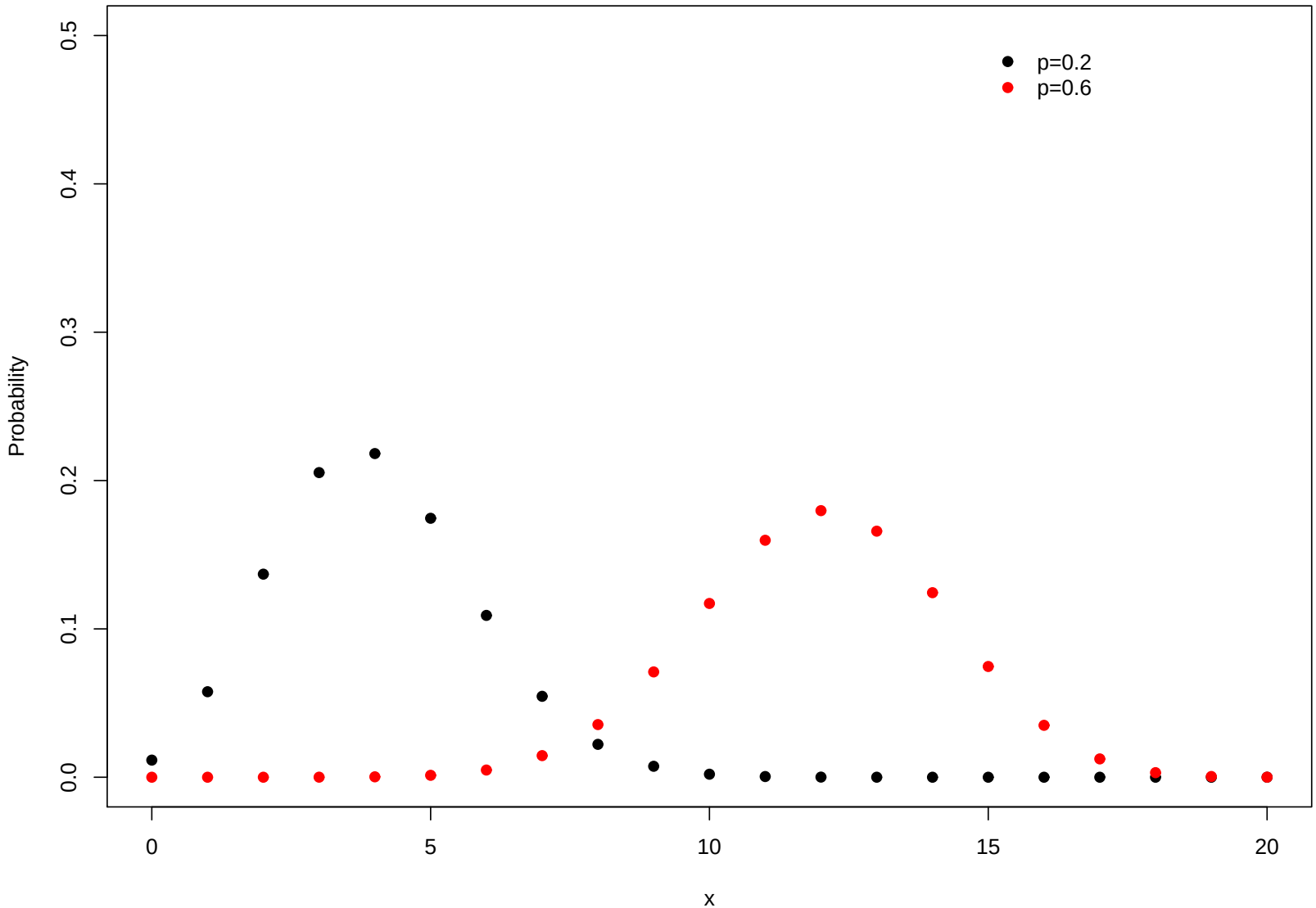
$n = 20, p = 0.5$



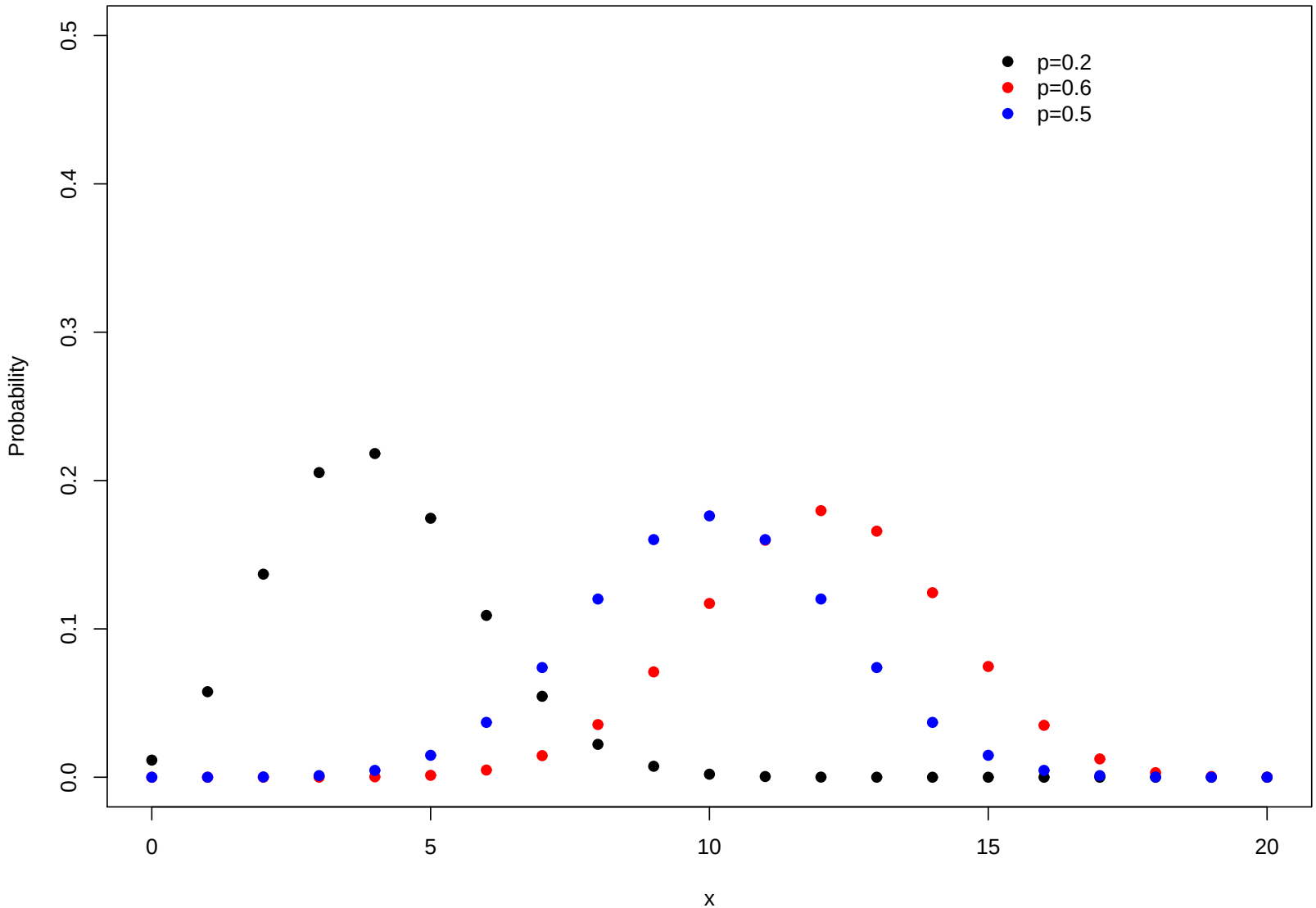
Comparison



Comparison



Comparison



$f(x)$ and $F(x)$

		0	1	2	3	4	5	6	7	8	9	10
$p = 0.2$	$f(x)$	0.012	0.058	0.137	0.205	0.218	0.175	0.109	0.055	0.022	0.007	0.002
	$F(x)$	0.012	0.069	0.206	0.411	0.630	0.804	0.913	0.968	0.990	0.997	0.999
$p = 0.6$	$f(x)$	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.015	0.035	0.071	0.117
	$F(x)$	0.000	0.000	0.000	0.000	0.000	0.002	0.006	0.021	0.057	0.128	0.245
$p = 0.5$	$f(x)$	0.000	0.000	0.000	0.001	0.005	0.015	0.037	0.074	0.120	0.160	0.176
	$F(x)$	0.000	0.000	0.000	0.001	0.006	0.021	0.058	0.132	0.252	0.412	0.588

		11	12	13	14	15	16	17	18	19	20
$p = 0.2$	$f(x)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$F(x)$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$p = 0.6$	$f(x)$	0.160	0.180	0.166	0.124	0.075	0.035	0.012	0.003	0.000	0.000
	$F(x)$	0.404	0.584	0.750	0.874	0.949	0.984	0.996	0.999	1.000	1.000
$p = 0.5$	$f(x)$	0.160	0.120	0.074	0.037	0.015	0.005	0.001	0.000	0.000	0.000
	$F(x)$	0.748	0.868	0.942	0.979	0.994	0.999	1.000	1.000	1.000	1.000