# Summary of the Non-Parametric Tests

- **Chi-Squared Test :** Goodness of Fit/independence in contingency tables
- **Sign Test :** One Sample (equivalent of one sample $t$-test)
- **Mann-Whitney-Wilcoxon :** Two Sample (equivalent of two sample $t$-test)
- **Wilcoxon Signed Rank :** Paired Data
- **Kruskal-Wallis :** one-way layout, multigroup comparison - equivalent of ANOVA for CRD.
- **Friedman :** two-way blocked layout, equivalent of two-way ANOVA for RBD.

**Pros:**

- No distributional assumptions
- Applicable for most sorts of data
- Large sample approximations make them easy to implement

**Cons:**

- Low power compared to parametric tests (i.e. often do not reject $H_0$ when they should - prone to Type II Error)
- Small sample null distributions difficult to compute.

# 3.6 Rank Correlation



To measure the association between two variables, we previously used the *correlation coefficient*, $r$; for data $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$,

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$$

where

$$SS_{xy} = \sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y}) \quad SS_{xx} = \sum_{i=1}^{n}(x_i - \overline{x})^2 \quad SS_{yy} = \sum_{i=1}^{n}(y_i - \overline{y}$$

$r$ is a measure of the linear association between $X$ and $Y$

**Pearson Product Moment Coefficient of Correlation**

A more general measure of association is the

**Spearman Rank Correlation Coefficient**

We compute this as follows:

1. For each sample separately, compute the **ranks** of the data, denote the ranks for the data $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ by $u_1, \ldots, u_n$ and $v_1, \ldots, v_n$ respectively.
2. Compute
$$r_S = \frac{SS_{uv}}{\sqrt{SS_{uu}SS_{vv}}}$$
ie the Pearson correlation between the ranks.

$r_S$ is the **Spearman Correlation**.

Notes:

1. If there are no ties in the data

$$r_S = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}$$

   where $d_i = u_i - v_i$.

2. $r_S$ is potentially a measure of the **non-linear** association between $X$ and $Y$.

   The calculation can be applied directly to rank data i.e. $u_1, \ldots, u_n$ and $v_1, \ldots, v_n$ can be preference ranks given by two observers.

# Tests for $r_S$

To test

$$H_0 \ : \ \rho = 0$$

vs

$$\begin{aligned} (1) \quad H_a &: \quad \rho > 0 \\ (2) \quad H_a &: \quad \rho < 0 \\ (3) \quad H_a &: \quad \rho \neq 0 \end{aligned}$$

We may use $r_S$ as a test statistic. The distribution of $r_S$ under $H_0$ is tabulated on p 864 of McClave and Sincich.

If $\text{Spearman}_\alpha$ is the $\alpha$ tail quantile of the null distribution, we have the following rejection regions:

$$
\begin{aligned}
(1) &\quad : \quad \text{Reject } H_0 \text{ if } r_S > \text{Spearman}_\alpha \\
(2) &\quad : \quad \text{Reject } H_0 \text{ if } r_S < -\text{Spearman}_\alpha \\
(3) &\quad : \quad \text{Reject } H_0 \text{ if } |r_S| > \text{Spearman}_{\alpha/2}
\end{aligned}
$$