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Simple Linear
Regression
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Multiple Linear Regression

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In the following tables columns are:

Complete Model

Reduced Model

SSE_C

SSE_R

k

g

F (test statistic)

F_{0.05}(k - g, n - k - 1)
```

We denote the critical value by F_{α} and check whether $F > F_{\alpha}$.

Potato Damage Data: ANOVA-F Tests

Simple Linear Regression

Multiple Linear Regression We compare four models: M_{R_1}, M_{R_2} and M_{R_3} are nested within the complete model M_C .

M_C	:	A + B + C + A.B + A.C + B.C + A.B.C
M_{R_1}	:	A + B + C + A.B
M_{R_2}	:	A + B + C
M_{R_3}	:	A + B + A.B

COMP.	RED.	SSE_C	SSE_R	k	g	F	F_{lpha}
M _C	M_{R_1}	4968.876	5093.746	7	4	0.561	2.76
M_{R_1}	M_{R_2}	5093.746	7183.674	4	3	28.721	3.92
M_{R_1}	M_{R_3}	5093.746	6319.640	4	3	16.846	3.92

Note: The quoted F_{α} values are approximate as the textbook does not tabulate all Fisher-F distributions. We take $\alpha = 0.05$

Multiple Linear Regression

Conclusions

Taking the comparisons in order:

 M_C vs M_{R1} : F < F_α. Therefore the result is not significant: Model M_{R1} is an adequate simplification of Model M_C, and we choose M_{R1} over M_C.

The model M_{R_1} now becomes the complete model.

- 2. $M_{R_1} vs M_{R_2} : F > F_{\alpha}$. Therefore the result is significant: Model M_{R_2} is not an adequate simplification of Model M_{R_1}
- 3. $M_{R_1} vs M_{R_3} : F > F_{\alpha}$. Therefore the result is significant: Model M_{R_3} is not an adequate simplification of Model M_{R_1}

Multiple Linear Regressior

Thus the final model is

$$A + B + C + A.B$$

i.e. all main effects, plus the interaction between potato variety and acclimatization routine.

We cannot simplify this model further without significant loss in terms of goodness of fit.

Note: $R^2 = 0.631$ and Adjusted $R^2 = 0.610$, so we have a reasonable fit.

Task Distraction Data

Simple Linear Regression

Multiple Linear Regression

Example: Task Distraction Data.

In an experimental study, the number of errors made in performing a specified task was recorded. The experiment investigated the influence of various predictors on the numbers of errors made.

There are two factor predictors (A, B) and one continuous covariate (X).

We have a balanced design with 15 people (replicates) in each factor-level subgroup.

Multiple Linear Regression

Example:	Task	Distraction Data.
A	Group	1 : Non-smoker
		2 : Delayed smoker
		3 : Active smoker
В	Task	1 : Pattern Recognition 2 : Cognitive Task 3 : Driving Simulation

X Distraction Level

Multiple Linear Regression We compare four models with the **complete** model

Complete Model : A * B * X

A + B + X + A.B + A.X + B.X + A.B.X

Number of parameters

Term	Parameters		
A	(a - 1)	= 3 - 1	2
В	(b-1)	= 3 - 1	2
Х	(1)		1
A.B	(a - 1)(b - 1)	$= 2 \times 2$	4
A.X	(a-1)(1)	$= 2 \times 1$	2
B.X	(b-1)(1)	$= 2 \times 1$	2
A.B.X	(a-1)(b-1)(c-1)	$= 2 \times 2 \times 1$	4
Total			17

Multiple Linear Regression For illustration we consider the following sequence of models: • Reduced Model 1: M_{R_1}

A + B + X + A.X + B.X

► Reduced Model 2: *M*_{*R*₂}

A + B + X + B.X

Reduced Model 3: M_{R3}

$$B + X + B.X$$

Reduced Model 4: M_{R4}

B + X

Task Distraction Data: ANOVA-F Tests

Simple Linear Regression

Multiple Linear Regression

M_C	:	A + B + X + A.B + A.X + B.X + A.B.X
M_{R_1}	:	A + B + X + A.X + B.X
M_{R_2}	:	A + B + X + B.X
M_{R_3}	:	B + X + B.X
M_{R_4}	:	B + X

COMP.	RED.	SSE_C	SSE_R	k	g	F	F_{α}
M_C	M_{R_1}	5660.010	7627.479	17	9	5.084	2.02
M_{R_1}	M_{R_2}	7627.479	7971.274	9	7	2.817	3.07
M_{R_2}	M_{R_3}	7971.274	8404.654	7	5	3.452	3.07
M_{R_3}	M_{R_4}	8404.654	11154.715	5	3	21.105	3.07

Conclusions

Simple Linear Regression

Multiple Linear Regression Taking the comparisons in order:

- 1. $M_C \text{ vs } M_{R_1} : F > F_{\alpha}$. Therefore the result is significant: Model M_{R_1} is not an adequate simplification of Model M_C
- 2. $M_{R_1} vs M_{R_2} : F < F_{\alpha}$. Therefore the result is not significant: Model M_{R_2} is an adequate simplification of Model M_{R_1}
- 3. $M_{R_2} vs M_{R_3} : F > F_{\alpha}$. Therefore the result is significant: Model M_{R_3} is not an adequate simplification of Model M_{R_2}
- 4. $M_{R_3} vs M_{R_4} : F > F_{\alpha}$. Therefore the result is significant: Model M_{R_4} is not an adequate simplification of Model M_{R_3}

Follow-up Analysis

Simple Linear Regression

Multiple Linear Regressior

In a follow up analysis (see Handout), it transpires that the model $% \left({{{\left[{{{\rm{T}}_{\rm{T}}} \right]}_{\rm{T}}}} \right)$

$$A + B + X + A.B + A.X + B.X$$

ie selected.

Note: $R^2 = 0.863$ and Adjusted $R^2 = 0.831$, so we have a good fit.

Note: we must take great care with the sequence of models.

Multiple Linear Regression

It is possible to carry out

Stepwise Selection in SPSS

- Forward
- Backward
- Stepwise

model selection in SPSS using the *Linear Regression* pulldown menu, and the *Method* pulldown list.

SPSS Screen for Stepwise Selection

Simple Linear Regression

Multiple Linear Regression

Linear Regres	ssion	
Age in years [age]	Dependent:	OK
 Weight (kg) [weight] Body mass percentage Forced expiratory volu Residual volume [rv] Functional residual caj 	Block 1 of 1 Previous Independent(s):	<u>R</u> eset Cancel Help
Total lung capacity [tlc Sex of patient [sex]	Body mass percentage	
	Method: Enter	
	Statistics Plots Save Optio	ns