Testing for Equal Treatment Means

An ANOVA F-test can be constructed for a RBD. Let

- $i = 1, \ldots, k$ index **treatments**
- $j = 1, \ldots, b$ index **blocks**

i.e. x_{ij} is the response for the ith treatment in the jth block. Let

- $\overline{x}_i$ be the ith **treatment** mean
- $\overline{x_j^{(B)}}$ be the jth **block** mean
- $\overline{x}$ be the **overall** mean

Let

$$SST = \sum_{i=1}^{k} b(\overline{x}_i - \overline{x})^2$$

$$SSB = \sum_{j=1}^{b} k(\overline{x_j^{(B)}} - \overline{x})^2$$

$$SS = \sum_{i=1}^{k} \sum_{j=1}^{b} (x_{ij} - \overline{x})^2$$

SST: Sum of Squares for **Treatments**
SSB: Sum of Squares for **Blocks**
SS: **Total** Sum of Squares

Finally

$$SS = SST + SSB + SSE \quad \therefore \quad SSE = SS - SST - SSB$$

SSE: Sum of Squares for **Errors**

Test statistic is

$$F = \frac{MST}{MSE}$$

where

$$MST = \frac{SST}{k-1} \qquad MSE = \frac{SSE}{n-b-k+1}$$

ANOVA F-test to compare treatment means in a randomized block design

Theorem: ANOVA F-test for a RBD.
To test

$$\begin{aligned} H_0 &: \mu_1 = \cdots = \mu_k \\ H_a &: \text{At least one pair of treatment means different.} \end{aligned}$$

use the test statistic

$$F = \frac{MST}{MSE}$$

If H_0 *is* **TRUE**

$$F \sim \text{Fisher-F}(k-1, n-b-k+1)$$

- this defines the rejection region for significance level α*, and the p-value, in the usual way.*

Assumptions:

1. Experimental units (between blocks) are independent, and treatments are allocated at random (within blocks).
2. Normality
3. bk block/treatment combinations correspond to populations with equal variances.

ANOVA Table

SOURCE	DF	SS	MS	F
TMTS	$k-1$	SST	MST	F = MSE/MSB
BLOCKS	$b-1$	SSB	MSB	
ERROR	$n-k-b+1$	SSE	MSE	
TOTAL	$n-1$	SS		

After the ANOVA test is complete, and the hypothesis

$$H_0 : \mu_1 = \cdots = \mu_k$$

is **rejected**, we can proceed with the “post-hoc” tests of hypotheses $\mu_i = \mu_j$ for $i \neq j$.

Notes:

1. In a RBD, it is not (in general) possible to estimate individual treatment means, that is, $\overline{x}_i$ does not estimate μ_i as it is an average across blocks, which are believed to be different in terms of response.

2. **Testing the Block Means** However, we can test whether the block means $\mu_1^{(B)}, \ldots, \mu_b^{(B)}$ are significantly different. For

$$H_0 : \mu_1^{(B)} = \cdots = \mu_b^{(B)}$$

we use the F statistic

$$F = \frac{MSB}{MSE}$$

where

$$MSB = \frac{MSB}{b-1}$$

If H_0 is **TRUE**

$$F \sim \text{Fisher-F}(b-1, n-k-b+1)$$

That is, we treat the blocks as levels of another factor, and test to see whether this factor affects response.

Example: Soil Analysis (see handout).

Results of two ANOVA F-tests:

Test of	F	p	Conclusion
SOLVENT	0.673	0.585	No Difference
SOIL	10.568	0.001	Difference

Here SOLVENT is the **blocking** variable, SOIL is the **treatment** variable.

3. Remember to check the assumptions (independence, normality, equal variances in each treatment/block combination)

   Equal variances may be hard to check as we only have one observation per treatment/block comparison.