MATH 204 - SOLUTIONS 1

1. We have

$$
\begin{aligned}
SS = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij}-\overline{x})^2 &= \sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij}-\overline{x}_i+\overline{x}_i-\overline{x})^2 \\
&= \sum_{i=1}^{k}\left\{\sum_{j=1}^{n_i}(x_{ij}-\overline{x}_i)^2+2(x_{ij}-\overline{x}_i)(\overline{x}_i-\overline{x})+(\overline{x}_i-\overline{x})^2\right\} \\
&= \sum_{i=1}^{k}\left\{\sum_{j=1}^{n_i}(x_{ij}-\overline{x}_i)^2\right\}+2\sum_{i=1}^{k}\left\{\sum_{j=1}^{n_i}(x_{ij}-\overline{x}_i)(\overline{x}_i-\overline{x})\right\} \\
&\quad +\sum_{i=1}^{k}\left\{\sum_{j=1}^{n_i}(\overline{x}_i-\overline{x})^2\right\}
\end{aligned}
$$

Now

$$
\begin{aligned}
\sum_{i=1}^{k}\left\{\sum_{j=1}^{n_i}(x_{ij}-\overline{x}_i)^2\right\} &= \sum_{i=1}^{k}(n_i-1)s_i^2=(n-k)s_P^2=SSE \\
\sum_{i=1}^{k}\left\{\sum_{j=1}^{n_i}(\overline{x}_i-\overline{x})^2\right\} &= \sum_{i=1}^{k}n_i(\overline{x}_i-\overline{x})^2=SST
\end{aligned}
$$

and

$$
\sum_{j=1}^{n_i}(x_{ij}-\overline{x}_i)(\overline{x}_i-\overline{x})=(\overline{x}_i-\overline{x})\sum_{j=1}^{n_i}(x_{ij}-\overline{x}_i)=(\overline{x}_i-\overline{x})\left\{\sum_{j=1}^{n_i}x_{ij}-n_i\overline{x}_i\right\}=0.
$$

Hence

$$
SS = SST + SSE
$$

as required.

2. The data yield the following statistics:

Treatment		0	2	4
Sample Size	n_i	4	5	5
Sample Mean	$\overline{x}_i$	26.75	33.60	38.20
Sample Variance	s_i^2	29.92	20.30	22.70

and

$$\begin{aligned} SST &= 292.1071 \\ SSE &= 258.7500 \\ SS &= 550.8571 \end{aligned}$$

so that

$$\begin{aligned} MST &= \frac{SST}{k-1} = \frac{292.1071}{2} = 146.054 \\ MSE &= \frac{SSE}{n-k} = \frac{258.7500}{11} = 23.523 \end{aligned}$$

and

$$F = \frac{MST}{MSE} = \quad = 6.209.$$

We compare this with the $1 - \alpha = 0.95$ point of the Fisher-F$(k-1, n-k)$ = Fisher-F$(2, 11)$ distribution. From tables (*McClave and Sincich*, p. 901)

$$F_\alpha(2, 11) = 3.98$$

and hence we

REJECT H_0

at the $\alpha = 0.05$ significance level.