MATH 204 - MID-TERM : SOLUTIONS

1. This is a **randomized block design** with replication; the variable **method** provides the treatment of interest, and **variety** is a **blocking** factor.

   For the analysis, we carry out an **ANOVA with interaction** using Levene's test to assess whether the variances are equal. The results are:

   Levene's Test of Equality of Error Variances[a]

   Dependent Variable: dry matter yield

F	df1	df2	Sig.
.920	14	75	.542

   Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

   a. Design: Intercept+method+variety+method * variety

   so the test does not reject the assumption of equal variances ($p = 0.542$). For the ANOVA

   Tests of Between-Subjects Effects

   Dependent Variable: dry matter yield

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1339.025[a]	14	95.645	4.867	.000
Intercept	30595.648	1	30595.648	1557.013	.000
method	953.156	2	476.578	24.253	.000
variety	11.380	4	2.845	.145	.965
method * variety	374.488	8	46.811	2.382	.024
Error	1473.767	75	19.650		
Total	33408.440	90			
Corrected Total	2812.792	89			

   a. R Squared = .476 (Adjusted R Squared = .378)

   From this table, we deduce

   - The treatment factor **method** is highly significant ($F = 24.253, p < 0.001$), so the hypothesis of equal method effects is **rejected**, and there is a significant effect of method on yield.
   - The blocking factor **variety** is not significant ($F = 0.145, p = 0.965$). Thus the apparent blocking factor does not seemingly give rise to different responses, contrary to our initial beliefs.
   - The interaction between **method** and **variety** has an associated p-value of 0.024. This is significant at $\alpha = 0.05$, but not at $\alpha = 0.01$. Thus the blocking factor appears to influence the response mildly through the interaction term.

   Thus it appears that there is definitively a difference between methods, but only questionably an influence of the blocking factor. Thus we could legitimately report the most appropriate model as

   method

   or

   method + **variety** + **method**.**variety**

However, the model

method + **method.variety**

should not be reported, as it is not a legitimate model.

Note that boxplots can verify that the ANOVA assumption of Normality is also met.

10 Marks

For a secondary analysis, we could refit the model without the variety variable and use a one-way ANOVA, or omit the interaction and re-fit. The results of a one-way ANOVA inform us as to which is the optimal method

Parameter Estimates

Dependent Variable: dry matter yield

					95% Confidence Interval	
Parameter	B	Std. Error	t	Sig.	Lower Bound	Upper Bound
Intercept	16.607	.844	19.674	.000	14.929	18.284
[method=1]	6.403	1.194	5.364	.000	4.031	8.776
[method=2]	-.910	1.194	-.762	.448	-3.283	1.463
[method=3]	0[a]	.	.	.	.	.

a. This parameter is set to zero because it is redundant.

growth method

Dependent Variable: dry matter yield

			95% Confidence Interval	
growth method	Mean	Std. Error	Lower Bound	Upper Bound
a	23.010	.844	21.332	24.688
b	15.697	.844	14.019	17.374
c	16.607	.844	14.929	18.284

From the parameter estimates table, the baseline category is growth method 3, so the mean level for that category is reported as *Intercept*. The other parameter estimates are *contrasts*, and show that the difference in mean response between method 1 and method 3 is 6.403, and between method 2 and method 3 is -0.910. Thus it appears that method 1 is the best method to use to maximize yield.

2. This is a **factorial design** with replication; the variables **operator** and **machine** are the factors of interest, and are to be treated equivalently in the analysis.

   (a) For the analysis, we carry out an **ANOVA with interaction** using Levene's test to assess whether the variances are equal. The results are shown in the table below. The test does not reject the assumption of equal variances ($p = 0.704$).

   For the ANOVA, see the table below. From this table, we deduce

   - The treatment factor **operator** is highly significant ($F = 23.253, p < 0.001$), so there is a significant effect of operator on yield.
   - The treatment factor **machine** is not significant ($F = 0.026, p = 0.994$). Thus there is no significant difference between machines.
   - The interaction between **operator** and **machine** is also not significant ($F = 0.157, p = 0.987$).

Thus it appears that there is definitively a difference between operators, but that no other factors or interactions induce a change in response. Hence the model to be reported is

operator

and no other factors should be included.

Note that boxplots can verify that the ANOVA assumption of Normality is also met.

Levene's Test:

Levene's Test of Equality of Error Variances[a]

Dependent Variable: Tensile Strength

F	df1	df2	Sig.
.733	11	84	.704

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept+operator+machine+operator * machine

ANOVA Table:

Tests of Between-Subjects Effects

Dependent Variable: Tensile Strength

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	986.333[a]	11	89.667	4.412	.000
Intercept	16432.667	1	16432.667	808.637	.000
operator	965.646	2	482.823	23.759	.000
machine	1.583	3	.528	.026	.994
operator * machine	19.104	6	3.184	.157	.987
Error	1707.000	84	20.321		
Total	19126.000	96			
Corrected Total	2693.333	95			

a. R Squared = .366 (Adjusted R Squared = .283)

15 Marks

For a secondary analysis, we could refit the model with just the variable operator included. The results of a one-way ANOVA inform us as to who is the best operator (see tables below)

From the parameter estimates table, the baseline category is operator 3, so the mean level for that category is reported as *Intercept*. The other parameter estimates are *contrasts*, and show that the difference in mean response between operator 1 and operator 3 is -7.656, and between operator 2 and operator 3 is -2.688.

(b) If the strips are known or strongly believed to be different, then **strip** should be fitted as a blocking factor. That is, we should attempt a three-factor analysis, with two treatment factors (**operator** and **machine**) and one blocking factor (**strip**). For this analysis, we **cannot fit interaction** as we do not have sufficient replications.

It could also be argued that we have already discovered that **machine** is not significant, so perhaps a two-factor RBD analysis with treatment factor **operator** and blocking factor **strip** could be used.

5 Marks

Parameter Estimates

Dependent Variable: Tensile Strength

Parameter	B	Std. Error	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	16.531	.762	21.696	.000	15.018	18.044
[operator=1]	-7.656	1.078	-7.105	.000	-9.796	-5.516
[operator=2]	-2.688	1.078	-2.494	.014	-4.827	-.548
[operator=3]	0[a]	.	.	.	.	.

a. This parameter is set to zero because it is redundant.

Operator

Dependent Variable: Tensile Strength

Operator	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
O1	8.875	.762	7.362	10.388
O2	13.844	.762	12.331	15.357
O3	16.531	.762	15.018	18.044