Jim Lambek, The Lorentz Category in Six Dimensions

Spencer Breiner, A scheme construction for logic and model theory

In this talk I will define an (affine) "logical scheme" and describe a representation theorem for logical theories which mimics Grothendieck's construction of affine schemes from commutative rings. The "spectral space" in our construction is a topological groupoid constructed from the semantics of the theory (models and isos). The "structure sheaf" (viewed as an etale space over the spectrum) is constructed from the definable sets of the models, on which the groupoid acts in a natural way. As with "algebraic" schemes we can recover the theory by taking (iso-stable) global sections of the structure sheaf. If time allows I will also give a few examples of non-affine schemes. Joint work by Spencer Breiner and Steve Awodey.

Gabor Lukacs, Algebraic invariants of topological structures

Final topological structures, such as quotients, tend to be far more complicated than initial ones, and they are certainly more difficult to describe. For example, the finest group topology on an abelian group in which a given sequence converges to zero is not metrizable (cf. [2]), and the uniform quotient of a metric space can have uniform weight as large as $\mathfrak{d}$ (cf. [1]).

The goal of this talk is to present an algebraic approach to topological structures that not only explains the underlying reason for the complexity of final structures, but also provides a concrete technique for describing them. Specifically, the lattice of uniformities and abelian group topologies on a given set or group, respectively, can be viewed as the lattice of idempotent ideals in a suitable ordered monoid; final structures are precisely the largest idempotent ideals contained in certain order ideals. This approach also allows us to establish upper bounds for the cardinal invariants of final structures.

References

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Pieter Hofstra, Isotropy quotients of toposes

Every (Grothendieck) topos contains a canonical group object called its isotropy group. This group has the property that it induces a so-called crossed topos structure, which in turn is a topos-theoretic generalization of the notion of a crossed module. In fact, the isotropy group is characterized by the fact that it gives rise to the terminal crossed topos. We may form the quotient of a topos by its isotropy group, which is a connected atomic quotient of the topos. Such a quotient is referred to as the isotropy quotient of the topos. In this talk, we explain what these quotients look like, discuss some special cases of interest and have a glimpse at higher-order isotropy groups, which arise when the isotropy quotient of a topos still has non-trivial isotropy. Joint work with Jonathon Funk and Benjamin Steinberg.

Ettore Aldrovandi, Butterflies and morphisms of monoidal and bimonoidal stacks

Morphisms between monoidal stacks, when the monoidal structure is group-like, are computed by certain diagrams of group objects called butterflies, owing to their shape. If presentations for the stacks are chosen, butterflies can be seen as forming the derived mapping spaces between length-one complexes of non necessarily abelian groups. In the abelian case this reproduces a classical result by Deligne (SGA 4 XVIII) characterizing the derived category in terms of morphisms of Picard stacks. There are interesting applications to the long exact sequence and to the change of coefficients in nonabelian cohomology. In particular the characteristic class of a group-like stack, which is classically that of a presentation by a crossed module, can be immediately obtained from the Postnikov decomposition of the stack. The classification by a degree 3 cohomology class is immediately obtained as a byproduct of the representation of morphisms by butterflies, without the need of generating an equivalence relation using free crossed modules. More recently, there has been an increasing interest in categories with two monoidal structures, in particular rig categories and categorical rings, for example because of connections with algebraic K-theory and elliptic cohomology. We can characterize the bimonoidal structure on a stack as being ring-like if the underlying stack is Picard and it is a monoid in the 2-category of Picard stacks. We show that stacks equipped with a ring-like structure can be presented by crossed bimodules, and morphisms between them can again be computed using butterfly diagrams, this time comprising ring objects and bimodules over them. Whereas the prototype of a butterfly in the context of group-like stacks is a group extensions, here it would be a non-necessarily abelian extension of rings. Thus the relevant results relate to the cohomology of rings in the sense of Shukla and MacLane, rather than group cohomology. Considering modules (say, right modules) for a ring-like stack, and in particular invertible ones, leads to a cohomology object which ought to be considered as a nonabelian version of those cohomologies: it can be described (albeit imprecisely) as a Shukla cohomology with values in a crossed bimodule. Then one can obtain classification results analogous to the one mentioned above for the case of a single monoidal structure.

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Rory Lucyshyn-Wright, A general Fubini theorem for the Riesz paradigm

We prove an abstract Fubini-type theorem in the context of monoidal and enriched category theory, and as a corollary we establish a Fubini theorem for integrals on arbitrary convergence spaces that generalizes (and entails) the classical Fubini theorem for Radon measures on compact Hausdorff spaces. For a symmetric monoidal closed adjunction $F \dashv G : \mathscr{L} \to \mathscr{X}$, we formulate a Fubini theorem as the statement that an associated *monad of natural distributions* $\mathbb{D} = \mathscr{L}([-,R],R)$ on $\mathscr{X}$ is *commutative* [2]. Under a mild completeness hypothesis on $\mathscr{L}$, we show that if each cotensor $[X,R]$ of the unit object R in $\mathscr{L}$ is reflexive (where $X \in \mathscr{X}$), then $\mathbb{D}$ is indeed commutative. For $\mathscr{X}$ the category of convergence spaces and $\mathscr{L}$ that of R-vector-spaces therein (for $R = \mathbb{R}$ or $\mathbb{C}$), a result of Butzmann [1] yields the needed reflexivity of the space $[X,R]$ of scalar functions, and a Fubini theorem for functionals $[X,R] \to R$ is thus obtained. This Fubini theorem extends immediately to *vector-valued* integration, our notion of which arises from $\mathbb{D}$ by analogy with [3].

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Mike Shulman, Categories, functors, and profunctors are a free cocompletion

Categories can be identified with monads in the bicategory of spans of sets; but similarly identifying functors, profunctors, and natural transformations is trickier. Ordinary morphisms of such monads include functors, but are too general; while ordinary monad 2-cells are not general enough. Drawing together work by Street, Lack, Carboni, Kasangian, Walters, Wood, and others, and using the language of weighted bilimits in enriched bicategories (which we have to develop for the purpose), we give a satisfactory solution to this problem. Namely, the "proarrow equipment" of categories, functors, and profunctors is the free cocompletion of the equipment of sets, functions, and spans under "tight Kleisli objects of loose monads", when both are regarded as a certain type of enriched bicategory – and likewise for enriched and internal categories. This is joint work with Richard Garner.

Noson Yanofsky, Galois Theory of Algorithms

Many different programs are the implementation of the same algorithm. The collection of programs can be partitioned into different classes corresponding to the algorithms they implement. This makes the collection of algorithms a quotient of the collection of programs. Similarly, there are many different algorithms that implement the same computable function. The collection of algorithms can be partitioned into different classes corresponding to what computable function they implement. This makes the collection of computable functions into a quotient of the collection of algorithms. Algorithms are intermediate between programs and functions:

$$\text{Programs} \Longrightarrow \text{Algorithims} \Longrightarrow \text{Functions}$$

Galois theory investigates the way that a subobject sits inside an object. We investigate how a quotient object sits inside an object. By looking at the Galois group of programs, we study the intermediate types of algorithms possible and the types of structures these algorithms can have.

Erwan Biland, Morita equivalences and strongly graded rings

Let G be any group and $R = 3D \bigoplus_{g \in G} R_g$ be a = strongly G-graded ring. The subring R_1 is endowed with an = additionnal structure, which can be characterized as an action of the = group G on the category of R_1-modules. This leads us to what we = call a G-equivariant ring. We study G-invariant modules and Morita = equivalences. We prove that any strongly G-graded algebra is Morita = equivalent to a G-interior algebra, along with other examples that = show the relevance of a functorial point on strongly graded rings.

Bob Pare, Multi-valued functors

Diers defines what it means for a functor to have a multi-adjoint but stops short of saying what sort of thing such a multi adjoint might be. Adamek and Rosicky as well as Makkai and Par use multi-limits but don't say where they live either. Lack and Street, in their paper on formal monads introduce the appropriate bicategory structure for Lawvere's partial functors but don't relate it to multi-adjoints. We put the two together and explore the consequences.

Dorette Pronk, Weakly globular double categories of fractions.

In this talk I will introduce the notion of a weakly globular double category. I will show how they model weak 2-categories by relaxing the globularity condition rather than the units or associativity axioms: instead of having a set of objects these weak 2-categories have a category of objects which is weakly equivalent to a discrete category, i.e., they have a posetal groupoid as objects. This is formalized by considering the posetal groupoid as the vertical arrow category in a (strict) double category.

As a first application of the rich structure that this notion of weak 2-category comes with I will introduce the weakly globular double category of fractions and its universal properties.

This is joint work with Simona Paoli.

Marta Bunge, Bounded Completions in Indexed Enriched Category Theory

Lawvere asked what are the common features of two known constructions, to wit, the Cauchy completion (Lawvere '73) and the stack completion (Bunge '79). The setting for our discussion is then necessarily a 2-Category $\mathbf{Cat}_(V, S)$ of 'S-indexed V-categories', such that $\mathbf{Cat}_(V, Set) = V - \mathbf{Cat}$ (Kelly '82) for V a symmetric monoidal closed category, and $\mathbf{Cat}_(S, S) = S$-Indexed $\mathbf{Cat}$ (Pare-Schumacher '78) for S a topos. A KZ-monad M on $\mathbf{Cat}_(V, S)$ is said here to be 'bounded' if (1) it is carved out from the Yoneda embedding, and (2) V has the defining property of M. Bounded KZ-monads that are completions, by which we mean here that they are idempotent, will be referred to as 'bounded completions'. The notion of a bounded completion unifies the Karoubi envelope, the stack completion, the Cauchy completion, and (what we call here) the Grothendieck completion. A bounded KZ-monad M on $\mathbf{Cat}_(V, S)$ is said here to be 'tightly bounded' if M satisfies a special form of Morita equivalence. Tightly bounded KZ-monads are completions. We show, by means of an equivalence between (adjoint) distributors and (adjoint) generalized functors, that the Cauchy and the Grothendieck KZ-monads, if considered in the same setting, are equivalent and tightly bounded. The Karoubi envelope is tightly bounded iff it agrees with the Cauchy completion, and the stack completion is tightly bounded iff it agrees with the Grothendieck completion.

Emily Riehl, Parametrized mates

In the presence of compatible adjunctions and functors, certain natural transformations involving the left adjoints correspond bijectively to certain natural transformations involving the right adjoints. Furthermore this *mates* correspondence is "natural"— more precisely it is "double functorial". A familiar consequence of these facts is that a lax T-algebra morphism admits a right adjoint iff it is strong. In this talk, presenting joint work with Eugenia Cheng and Nick Gurski, we generalize this situation to n-variable adjunctions, introducing *parametrized mates*, and characterize the "naturality" of this correspondence.