Spectra for Symbolic Dynamics

By a *flow* on a compact Hausdorff space *X*, we mean a map $t : X \to X$. Each point $x \in X$ traces out an *orbit* $\{x, t(x), t^2(x), ..., t^n(x) ...\}$. We consider questions such as whether the orbit eventually cycles or approaches a limit or behaves chaotically. Symbolic dynamics arise, for example, when $X = A_0 \cup A_1$ where A_0 and A_1 are closed subsets of *X*. Each $x \in X$ traces out one (or more) sequences $s = (s_0, ..., s_n, ...)$ of *symbols* (in this case 0's and 1's) for which $t^n(x) \in A_{s(n)}$. The set of all such sequences is a closed subspace of 2^N and this approximates the original flow by a flow on a compact Hausdorff, totally disconnected space. Each such space is determined by the Boolean algebra, *B*, of its clopen subsets, and the map *t* leads to a Boolean homomorphism $\tau = t^{-1} : B \to B$. Questions about cyclic orbits, etc., can be expressed in terms of τ and the Boolean operations on *B* and we can use topos theory (locales suffice) to construct the cyclic spectrum of the *Boolean flow* (B, τ). If $t : P \times X \to X$ is a parameterized flow, where *P* is the space of parameters, then the above construction produces a sheaf of Boolean flows over *P*, so we can again apply the theory of spectra. The extension to the case where $X = A_0 \cup A_1 \cup ... \cup A_{n-1}$ (so we have *n* symbols) is straightforward.