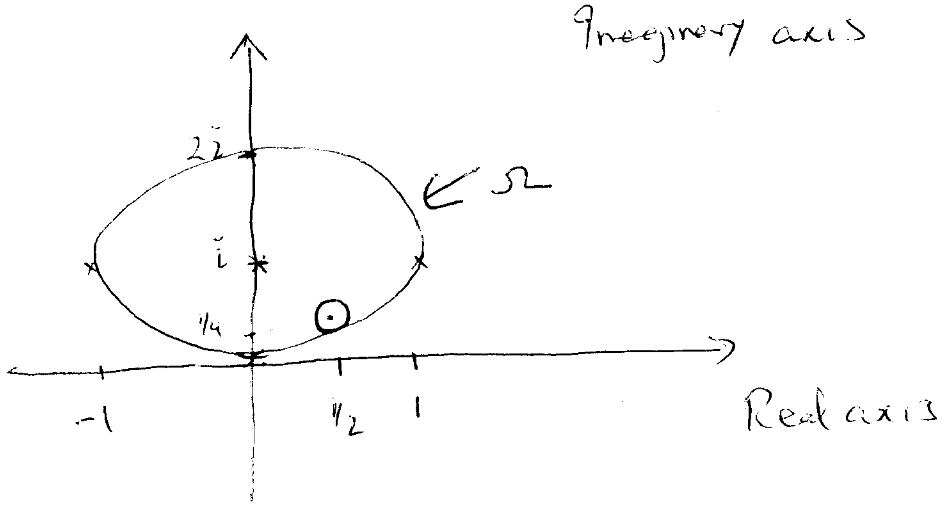
# MATH-381

## Homework # 1 Solutions

1.




$\left(\frac{1}{2}, \frac{i}{4}\right)$

Lets find the equation of circle and evaluate y-coordinate at $x=1/2$ //.

$$r = 1-\sqrt{(+1-1/4)^2+(0-1/2)^2}$$

$$\left|z-\left(\tfrac{1}{2}+i/4\right)\right| < r = \frac{4-\sqrt{13}}{4}$$

$$(x-x_0)^2+(y-y_0)^2=r^2$$

$$x^2+(y-1)^2=1$$

$$x=1/2 \Rightarrow (1/2)^2+(y-1)^2=1$$

$y = 1.866$ or $0.134$

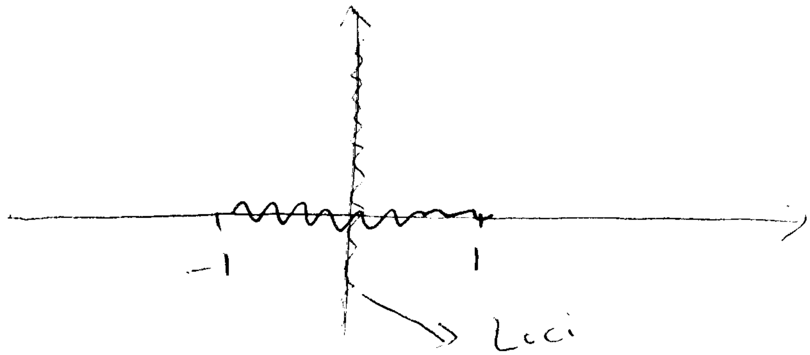
We choose $y=0.134$ ($1.866=y$ is not the closest coordinate)

radius of our loci $\Rightarrow$ $r = 1/4 - 0.134 = 0.116$

$$|z-(1/2+i/4)| < 0.116 \Rightarrow$$

$$|z-(1/2+i/4)| < 0.116$$

2.




~~$z = ni$ where~~

$z = n$ $\quad$ $-\cancel{\infty < n < +\infty}$

$-1 \le n \le 1$

3.

$$f(z) = \frac{z - i}{z^2 - 3iz - 2} = \frac{z - i}{(z^2 - 2iz - iz - 2)}$$

$$= \frac{z - i}{(z-i)(z-2i)} = \frac{1}{z - 2i}$$

$$\lim_{z \to i} f(z) = \frac{1}{i - 2i} = \frac{1}{-i} = i$$

$$f(i) = i = \lim_{z \to i} f(z) \Rightarrow f(z) \text{ is continuous @ } z = i$$

(4)

(a) $z = x + iy$

$f'(z)$ exists where C-R equations are satisfied

$z^2 = (x+iy)(x+iy) = x^2 + ixy + ixy - y^2$

$= (x^2 - y^2) + 2xyi$

$f(z) = \underbrace{(x-1)^2 + (x^2 - y^2)}_{u} + i\,\underbrace{(y^2 - 2xy)}_{v}$

$u_x = v_y \Rightarrow \quad 2(x-1) + 2x = 2y - 2x$

$2x - 2 + 2x = 2y - 2x$

$4x - 2 = 2y - 2x$

$6x = 2y + 2$

$3x = y + 1$

$\boxed{y = 3x - 1}$

$v_x = -u_y \Rightarrow \quad -2y = -2y$

$\boxed{y = 3x - 1}$ defines where $f'(z)$ exists

$g(z) = \underbrace{x}_{u} + i\,\underbrace{|y|}_{v}$

$\cancel{u_x = v_y}$

$\cancel{y \neq 0}$

$\cancel{1 \neq 1}$

$y > 0$

$g(z) = \underbrace{x}_{u} + i\underbrace{y}_{v}$

$u_x = v_y \Rightarrow 1 = 1$

$v_x = -u_y \Rightarrow 0 = 0$

$g'(z)$ exists for all $x, y$

$y < 0$

$g(z) = x - iy$

$u_x = v_y \Rightarrow 1 = -1$

Never exists

$g'(z)$ exists for $y > 0$ for all $x$ and $y$

5)

a) there is no discontinuity as the function is a polynomial. the function is differentiable everywhere and hence analytic everywhere. It is an entire function!!!

5)

b) $f'(z) = 3z^2 + 2z$

$$f'(1+i) = 3(1+i)(1+i) + 2(1+i)$$
$$= 3(1+i+i+i^2) + 2+2i$$
$$= 3(1+2i-1) + 2+2i = 6i+2+2i = \boxed{2+8i}$$

6)

a) $f(z) = (x+iy)(\cos x \cosh y - i \sin x \sinh y)$

$$f(z) = \underbrace{x\cos x\cosh y + y\sin x\sinh y}_{u} + i\,\underbrace{(-x\sin x\sinh y + y\cos x\cosh y)}_{v}$$

Entire function $\Rightarrow$ C-R satisfied everywhere

$u_x = \cos x\cosh y - x\sin x\cosh y + y\cos x\sinh y = -x\sin x\cosh y + \cos x\cosh y + y\cos x\sinh y = v_y$

$u_x = v_y$ ✓

$v_x = -u_y$ ✓

$-\sin x\sinh y - x\cos x\sinh y - y\sin x\cosh y = -x\cos x\sinh y - y\sin x\cosh y - \sin x\sinh y$

$\Rightarrow \left[\begin{array}{l} u_x = v_y \ \& \\ v_x = -u_y \end{array}\right.$ for all $z$ $\Rightarrow$ entire function !!!

(b)

$$f'(z) = u_x + i v_x$$

$$f'(z)\Big|_{z=i} = f'(i) \Rightarrow x = 0,\ y = 1$$

$$u_x \Big|_{x=0,\, y=1} = \cosh(1) + \sinh(1)$$

$$v_x \Big|_{x=0,\, y=1} = 0$$

$$f'(i) = \cosh(1) + \sinh(1)$$