

## Mathematics 381 Midterm Sample Problems

1) Let  $a$  be a real number with  $|a| > 1$ . Compute

$$\int_0^{2\pi} \frac{1 - a \cos \theta}{1 - 2a \cos \theta + a^2} d\theta.$$

Please justify your answer.

2) Let  $C$  be a simple closed contour and let  $z_1$  and  $z_2$  lie inside  $C$ . Compute

$$\frac{1}{2\pi i} \oint_C \frac{\sin z}{(z - z_1)(z - z_2)} dz.$$

Please justify your answer.

3) Let  $f(z) = e^x + ie^{2y}$  where  $z = x + iy$  is a complex variable defined in the whole complex plane. For what values of  $z$  does  $f'(z)$  exist?

4) (a) Show that  $u(x, y) = e^x \cos y + e^y \cos x + xy$  is harmonic.

(b) Compute the harmonic conjugate of  $u(x, y)$ .

5) Find  $\int_{1+i}^{-1-i} \frac{\text{Log } z}{z} dz$  where the integral is along a contour not intersecting the branch cut for  $\text{Log } z$ .

6) (a) Find the antiderivative of  $f(z) = \frac{1}{1+z^2}$ .

(b) Find the specific antiderivative that equals  $\frac{\pi}{4}$  when  $z = 1$ .