Mathematics 381 : Homework # 1

Due (in class): Wednesday, September 28.

1. Consider the open set $\Omega = \{z \in C; |z - i| < 1\}$. Write down an explicit formula (ie. an inequality in z) for a neighbourhood of $\frac{1}{2} + \frac{i}{4}$ contained entirely within Ω.

2. Let $\Omega \subset R^2$ be the set of points the sum of whose distances from the points $(1, 0)$ and $(-1, 0)$ equals 2. Represent Ω as a set in the complex plane, C, by means of equations or inequalities in $z \in C$.

3. Prove that the following function is continuous at $z = i$:

$$f(z) = \frac{z - i}{z^2 - 3iz - 2}, \quad z \neq i \quad and \quad f(i) = i.$$

4. (a) For what values of $z = x + iy \in C$ does the function $f(z) = (x - 1)^2 + iy^2 + z^2$ have a complex derivative? What about $g(z) = x + i|y|$?

(b) Compute $f'(z)$ and $g'(z)$ at all points at which the complex derivatives exist.

5. (a) Where is the function $f(z) = z^3 + z^2 + 1$ analytic?

(b) Find an expression for $f'(z)$ and compute $f'(1 + i)$.

6. (a) Show that $f(z) = z[\cos x \cosh y - i \sin x \sinh y]$ is an entire function where $z = x + iy$.

(b) Write down the formula for $f'(z)$ and compute $f'(i)$.