PHILATELIC LATIN SQUARES

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Abstract. Postage stamps are occasionally issued in sheetlets (small sheets, miniature sheets, souvenir sheets) of \(n\) different stamps printed in an \(n \times n\) array with \(n\) copies of each of the \(n\) stamps. We focus on sheetlets that form a philatelic Latin square (PLS): each of the \(n\) stamps appears exactly once in each row and exactly once in each column. We give examples for \(n = 2, 3, 4, 5\) building on results in \([5, 16, 17, 18, 30]\); see also \([29, 32]\).

Résumé. Des timbres sont parfois émis en planches (petites, miniatures, souvenirs) de \(n\) différents timbres imprimés dans un tableau \(n \times n\) avec \(n\) exemplaires de chacun des \(n\) timbres. Nous nous concentrons sur les planches qui forment un carré latin philatélique (PLS en anglais) : chacun des \(n\) timbres apparaît exactement une fois dans chaque ligne et exactement une fois dans chaque colonne. Nous donnons des exemples pour \(n = 2; 3; 4; 5\) en nous basant sur les résultats de \([5, 16, 17, 18, 30]\); voir aussi \([29, 32]\).

1. INTRODUCTION AND MISE-EN-SCÈNE

Postage stamps are occasionally issued in sheetlets (small sheets, miniature sheets, souvenir sheets) of \(n\) different stamps printed in an \(n \times n\) array with \(n\) copies of each of the \(n\) stamps. Here we focus on sheetlets where each of the \(n\) stamps appears exactly once in each row and exactly once in each column. We call such a sheetlet a philatelic Latin square (PLS) and have found PLS for \(n = 2, 3, 4, 5\); we would be pleased to learn of any PLS with \(n \geq 6\). For more about Latin squares see, e.g., \([6, 24]\).

The mathematics educator William Leonard Schaaf (1898–1992) in his Mathematics and Science: An Adventure in Postage Stamps \([25, p. xiii]\) observed that “The postage stamps of the world are, in effect, a mirror of civilization”, while Skaggs \([27]\) refers to postage stamps as “Icons of communication”. Moreover, “Philately is the study of stamps and is distinct from stamp collecting—it is possible to be a philatelist without owning any stamps!” \([37]\) and at this 2010 Annual Meeting of the CSHPM: “Mathematical philately is a liberal art!” \(^1\)

\(^{1}\) This paper is based on the talk presented (by George P. H. Styan) in the Special Session on Mathematics and the Liberal Arts at the Annual Meeting of the Canadian Society for the History and Philosophy of Mathematics, Concordia University, Montréal (Québec), Canada, May 31, 2010, and, in part, on \([5, 16, 17, 18, 30]\).
We were surprised to find that Latin square designs have been used in printing postage stamps, with the first such use apparently being in

- 1949 USSR 2 × 2: *Scott* 1359–1360 (Figure 2.1, left panel),
- 1960 Turkey 5 × 5: *Scott* 1488–1492 (Figure 5.2),
- 1972 Canada 4 × 4: *Scott* 582–585 (Figure 4.2, left panel),
- 1976 Isle of Man 3 × 3: *Scott* 86–88 (Figure 3.2, left panel).

Throughout this paper *Scott* catalog numbers are as found in the *Scott Standard Postage Stamp Catalogue* [26].

Latin square designs of size 4 × 4 have been used in Europe at least since the 13th century, when the Majorcan writer and philosopher Ramon Llull (1232–1316) published the “First elemental figure” in his *Ars Demonstrativa* [15] in 1283. As described by Preece [23], probably the earliest use of a Latin square in an experimental design was by the agronomist François Cretté de Palluel (1741–1798), who in 1788 published a study [7] of an experiment involving the fattening of 16 sheep in France, four of each of four different breeds. The advantage of using a Latin-square design in this experiment was that only 16 sheep were needed rather than 64 for a completely cross-classified design. The purpose here was to show that one might just as well feed sheep on root vegetables during the winter; this was much cheaper, and easier, than the normal diet of corn and hay.

2. PLS with \( n = 2 \)

There are many 2 × 2 PLS but the oldest we have identified is from the USSR (Figure 2.1, left panel) issued in 1949 in celebration of the 150th birth anniversary of Alexander Sergeyevich Pushkin (1799–1837), who is considered to be the greatest Russian poet [37]. The stamp in the top-left corner shows Pushkin in 1822 and the stamp in the top-right corner is based on a portrait by Orest Adamovich Kiprensky (1782–1836) painted in 1827. This is the oldest PLS (of any size) that we have identified to date.

The 2 × 2 PLS (Figure 2.1, right panel) was issued by Romania in celebration of the centenary of the Scouting Movement. The stamp on the top-left of the PLS shows its founder, Robert Stephenson Smyth Baden-Powell, 1st Baron Baden-Powell (1857–1941), whose father, Reverend Professor Baden Powell (1796–1860), was Savilian Professor of Geometry at Oxford [22]. This PLS was issued in the “Europa” series. Europa stamps have been issued annually since 1956; from 1959–1992 the Europa series stamps represented the European Conference of Postal and Telecommunications Administrations (CEPT), from 1993 to date the stamps represented PostEurop [sic], the

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2The acronym CEPT comes from the French: “Conférence européenne des administrations des postes et télécommunications”.


Association of European Public Postal Operators, one of the constituent unions of the Universal Postal Union. There are many $2 \times 2$ PLS issued in the Europa series and we recommend the McLean website [20].

![Image of Philatelic Latin Squares](image-url)

**FIGURE 1.3:** USSR 1949, Scott 1359–1360; (right) Romania 2007, Scott 4941–4942.

![Image of Philatelic Latin Squares](image-url)

**FIGURE 2.1:** USSR 1949, Scott 1359–1360; (right) Romania 2007, Scott 4941–4942.

![Image of Philatelic Latin Squares](image-url)

**FIGURE 2.2:** Isle of Man 1981, Scott 198–199; (right) Great Britain 2005, Scott 2279ab.
A 2 × 2 PLS (Figure 2.2, left panel) was issued by the Isle of Man in celebration of the marriage of The Prince Charles, Prince of Wales (b. 1948) to Lady Diana Spencer (1961–1997) on 29 July 1981 and a 2 × 2 PLS (Figure 2.2, right panel) was also issued by Great Britain in celebration of his marriage to Camilla Parker Bowles (b. 1947) on 9 April 2005. Camilla, Duchess of Cornwall is the current holder of the titles Princess of Wales and Duchess of Cornwall and of Rothesay [37]. The marginal inscription (in English and Welsh) states that the wedding took place on 8 April 2005, but it was delayed till the following day 9 April due to the Prince’s attendance at the April 8 funeral of Pope John Paul II (1920–2005).

The Isle of Man is a self-governing British Crown Dependency, located in the Irish Sea between the islands of Great Britain and Ireland, within the British Isles. The head of state is Queen Elizabeth II, who holds the title of Lord of Mann (sic). The island is not part of the United Kingdom, but its foreign relations and defence are the responsibility of the UK Government. Isle of Man stamps are indexed by Scott [26] under Great Britain.

2.1. 2×2 embedded-PLS.

Figure 2.3: Canada 2009, Scott 2326–2327.
There are many $2 \times 2$ PLS in sheetlets of 4 stamps but it seems there are even more $2 \times 2$ PLS embedded in sheetlets of $4k$ stamps, arranged “checkerboard-style”, with $k \geq 2$. We will refer to such sheetlets as embedded-PLS. For example, in the PLS issued by Canada in a series to “Preserve the Polar Regions and Glaciers” (Figure 2.3), $k = 4$ and embedded is a $2 \times 2$ PLS featuring a stamp with a polar bear ($Ursus maritimus$) on the main diagonal and a $2 \times 2$ PLS featuring a stamp with an Arctic Tern ($Sterna paradisaea$) on the main diagonal. There are in all in this sheetlet 5 copies of the PLS featuring the polar bear and 4 copies of the PLS featuring the arctic tern.

In Figure 2.4 we display two sheetlets issued for the World Wide Fund for Nature (WWF), an international non-governmental organization working on questions regarding the conservation, research and restoration of the environment, formerly called the World Wildlife Fund, which remains its official name in the United States and Canada. There are many PLS issued for the WWF and we recommend the websites of Groth AG [13] and Steurs [28].

The Viet Nam sheetlet (Figure 2.4, left panel) features the Saola or Vu Quang ox ($Pseudoryx nghetinhensis$), one of the world’s rarest mammals. It is a forest-dwelling bovine found only in Viet Nam and in Laos, near the Viet Nam–Laotian border. Rows 2 and 3 form a $2 \times 2$ PLS.

The other sheetlet (Figure 2.4, right panel) was issued by Turkmenistan (Mary Velayat) 1998 and features “wild animals and birds”. Mary Velayat is the southeastern region of Turkmenistan on the Turan Lowland, within the
limits of the Karakum desert. The top two rows, and the bottom two rows, each form two $2 \times 2$ PLS which are “block-reflections” of each other, and so embedded in this sheetlet are four $2 \times 2$ PLS. For stamps from countries and certain regions that were part of the USSR we recommend BiStamp [2].

2.2. $2 \times 2$ block-PLS.

Figure 2.5: Macau 2008, Scott 1248ah. (Joint issue with Singapore 2008.)

Figure 2.6: Singapore 2008, Scott 1318ad–1319ad. (Joint issue with Macau 2008.)
There is a $2\times2$ block-PLS entitled “Local gastronomy” from Macau (Figure 2.5), issued jointly with Singapore (entitled “Local delights”), where each block comprises 4 stamps, and so, 8 distinct stamps in all in a sheetlet of 16 stamps. The Latin square underlying a $2\times2$ block-PLS is a “block-Latin square”. We do not know if the Singapore stamps (Figure 2.6) were also issued as a $2\times2$ block-PLS and we welcome information about this.

Featured on the stamps are 4 Macau dishes: Carne de porco à Alentejana (Pork and clams), Lombo de bacalhau braseado em lascas (Grilled salt cod slices), Yangzhou fried rice, Crispy fried chicken and 4 Singapore dishes: Roti prata (fried flour-based pancake), Hainanese chicken rice, Laksa (Spicy noodle soup), Satay (Meat marinated in a spicy peanut sauce on a skewer).

Figure 2.7: Hong Kong 2003, Scott 1063–1068.
The only other $2 \times 2$ block-PLS we have found is from Hong Kong (Figure 2.7) issued for “World heritage in China”. Here each block comprises 3 stamps with the two blocks in the top “row-block” (of 6 stamps) repeated in the bottom row-block; the sheetlet, therefore, includes 2 distinct $2 \times 2$ block-PLS (block-reflections of each other) one being the top 2 rows and the other being the bottom 2 rows. The stamps feature (HK$1.40) Potala Palace, ($1.80) Imperial Palace of the Hing and Qing Dynasties, ($2.40) Mausoleum of the First Qin Emperor, ($2.50) Mount Huangshan, ($3.00) Old Town of Lijiang, ($5.00) Jiuzhaigou Valley.

3. PLS with $n = 3$

There are just two types of $3 \times 3$ PLS: one-step backwards circulant (C1B) as illustrated by the sheetlet depicting trees of the Holy Land issued by Israel (Figure 3.1, left panel); depicted are the Greek strawberry tree (*Arbutus andrachne*), Judas tree (*Cercis siliquastrum*), and Vallonea oak (*Quercus ithaburensis*). This PLS (Figure 3.1, right panel) is the one-step forwards circulant (C1F) issued by the “Grenadines of St. Vincent” in 1993 for Elvis Presley (1935–1977); a PLS with the same design was also issued in 1993 by Dominica, Gambia, Maldives, Nevis, and St. Vincent (*Scott* 1767). As of October 31, 2010, we have identified 45 (including 18 from Abkhazia) PLS with $n = 3$: only 8 of type C1B and the other 37 of type C1F, including 6 for Elvis Presley and 9 embedded (6 from Costa Rica and 3 from St. Vincent).

![Figure 3.1: Israel 1981, Scott 798–800; (right) “Grenadines of St. Vincent” 1993, Scott 1347ac (listed under “St. Vincent Grenadines”).](image-url)
Saint Vincent and the Grenadines is a nation which lies at the southern end of the eastern border of the Caribbean Sea where it meets the Atlantic Ocean. Its 389 km$^2$ (150 mi$^2$) territory consists of the main island of Saint Vincent and the northern two-thirds of the Grenadines, which are a chain of smaller islands stretching south from Saint Vincent Island to Grenada.

The oldest 3×3 PLS that we have identified is from the Isle of Man issued in 1976. Issued for the Europa CEPT programme, this 3×3 PLS (Figure 3.2, left panel) is one of a pair featuring Manx ceramic art: (top row, left to right) Barrosse beaker (Bronze Age), souvenir teapot (3-legged man, 19th century), Laxey jug (1854). Another pair of 3×3 PLS were issued in 1978 featuring carved gravestones: (Figure 3.2, right panel, top row, left to right) wheel-headed cross-slab, Celtic wheel-cross, Keeil Chiggyrt stone. All four 3×3 PLS issued by the Isle of Man in 1976/1978 are of the one-step forwards circulant (C1F) type.

![Figure 3.2: Isle of Man 1976, Scott 86–88 & (right) 1978, Scott 131–133.](image)

3.1. 3×3 embedded-PLS. Three distinct 3×3 PLS are embedded in the 4×4 sheetlet (Figure 3.3) issued by “St. Vincent & The Grenadines” for Sandy Koufax (b. Sanford Braun, 1935), the American left-handed former baseball pitcher in US Major League Baseball who played his entire career for the Brooklyn/Los Angeles Dodgers, from 1955–1966. All four 3×3 PLS are of the one-step forwards circulant (C1F) type with the PLS formed from the first 3 rows and columns repeated in the PLS formed from the last 3 rows and columns. We have also found six 3×3 PLS embedded in sheetlets from Costa Rica issued in 2005 (Scott 586ac, 596ac) depicting flora and fauna in national parks (3) and indigenous fruits (3).
4. PLS with \( n = 4 \)

There are just four \( 4 \times 4 \) Latin square matrices in so-called reduced-form, i.e., \( 1, 2, 3, 4 \) in the top row and in the first column:

\[
L_a = \begin{pmatrix}
1 & 2 & 3 & 4 \\
2 & 3 & 4 & 1 \\
3 & 4 & 1 & 2 \\
4 & 1 & 2 & 3 \\
\end{pmatrix}, \quad L_b = \begin{pmatrix}
1 & 2 & 3 & 4 \\
2 & 1 & 4 & 3 \\
3 & 4 & 1 & 2 \\
4 & 3 & 2 & 1 \\
\end{pmatrix}, \quad L_c = \begin{pmatrix}
1 & 2 & 3 & 4 \\
2 & 1 & 4 & 3 \\
3 & 4 & 2 & 1 \\
4 & 3 & 1 & 2 \\
\end{pmatrix}, \quad L_d = \begin{pmatrix}
1 & 2 & 3 & 4 \\
2 & 4 & 1 & 3 \\
3 & 1 & 4 & 2 \\
4 & 3 & 2 & 1 \\
\end{pmatrix}.
\]

Permuting the last three rows of each of the four Latin square matrices \( L_a, L_b, L_c, L_d \) generates 6 Latin squares, and so there are 24 so-called standard-form \( 4 \times 4 \) Latin squares in all. By standard-form we require that the top row be \( 1, 2, 3, 4 \) but not necessarily the first column.
We identify these 24 standard-form Latin squares with a subscript letter $a, b, c, d$ and a superscript set of three numbers as they appear in column 1, rows 2,3,4. For example we represent the PLS (Figure 4.1, left panel) of type $a234$ by

$$L_a^{(234)} = \begin{pmatrix}
1 & 2 & 3 & 4 \\
2 & 3 & 4 & 1 \\
3 & 4 & 1 & 2 \\
4 & 1 & 2 & 3
\end{pmatrix}.$$  

We present examples of 10 different types of $4 \times 4$ PLS from 8 different countries: Albania, Canada, Gambia, Guinea, Guinea-Bissau, Pitcairn Islands, Portugal, and Tristan da Cunha issued from 1972–2009.

As of October 31, 2010, we have identified 172 PLS with precisely 100 issued for the WWF 1988–2009 from 73 countries and 33 from Macau (31 of type $b324$) but only of these 10 types (out of 24): The most popular (by far) are:

Figure 4.1: Albania 1999, Scott 2590 (Mickey Mouse) & 2007, Scott 2811 (Pink Panther).
\[ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix}, \]

\[ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}, \]

\[ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 2 & 1 \end{bmatrix}. \]

The sheetlet in Figure 4.1 (left panel) forms a PLS of type \( a^{234} \), which is a one-step backwards circulant, and is \( 2 \times 2 \) block-Latin in that the top left \( 2 \times 2 \) corner of four stamps is the same as that in the bottom right corner, and the top right corner of four stamps is the same as that in the bottom left corner. This seems to be the most popular type of PLS—we have found 58 examples as of October 31, 2010. The stamps were issued by Albania in 1999 and feature Mickey Mouse.

The sheetlet in Figure 4.1 (right panel) forms a PLS of type \( a^{432} \), which is a one-step forwards circulant, which is also popular—we have identified 40 examples as of October 31, 2010. The stamps were issued by Albania in 1999 and feature the Pink Panther.

The matrix \( L^{324} \) and the array of 16 stamps in the Canada-1972 PLS (Figure 4.2, left panel) are in Sudoku form in that the four stamps each appear once in the top left, top right, bottom left, and bottom right \( 2 \times 2 \) corners, and so the full Latin square is a solution to a \( 4 \times 4 \) mini-Sudoku puzzle; Yates [39] says that such Latin squares have “balanced corners”. For more about Sudoku see, e.g., [1, 3, 4, 34, 35, 38].

The PLS in Figure 4.2 (right panel) is essentially the only one we have found of type \( a^{342} \), which is also in Sudoku form, like the PLS of type \( a^{324} \) from Canada (Figure 4.2, left panel). Depicted is the subantarctic fur seal (Arctocephalus tropicalis). The stamps were issued in 2004 by Tristan da Cunha, a group of islands in the south Atlantic.

The sheetlet shown in Figure 4.3 forms a PLS of type \( b^{324} \), which is in Sudoku form and forms a magic square; this type is very popular. This sheetlet was issued by Canada in 2005 and features 4 Canadian bridges as described in detail on the sheetlet-verso (Figure 4.4): The Jacques Cartier Bridge across the St. Lawrence River, the Angus L. Macdonald Bridge across Halifax Harbour, the Canso Causeway connecting the Nova Scotia mainland with Cape Breton Island and the Swinging Bridge in Souris, Manitoba.

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\(^3\)We have found 3 more PLS of type \( a^{342} \) embedded in larger sheets of stamps [16].
Figure 4.2: Canada 1972, Scott 582–585; (right) Tristan da Cunha 2004, Scott 747–750.

Figure 4.3: Canada 2005, Scott 2100–2103.

Figure 4.4: Canada 2005, Scott 2100–2103 (verso).
The PLS in Figure 4.5 (left panel), which is the only one of type $b_{342}$ that we have identified, is also in Sudoku form and also forms a magic square, this time with both diagonals “Latin” in that each stamp appears once in the main forwards and backwards diagonals. Depicted is the African buffalo ($Syncerus caffer$), a large African cloven-hoofed mammal.

The sheetlet in Figure 4.5 (left panel) was issued by Guinea-Bissau ($Guinée-Bissau$), in western Africa; formerly Portuguese Guinea, upon independence, it added the name of its capital, Bissau, to the country’s name in order to avoid confusion between itself and the nearby Republic of Guinea ($République de Guinée$), which was formerly French Guinea (and which issued the PLS of type $c_{423}$ shown in Figure 4.6).

This PLS of type $b_{423}$ (Figure 4.5, right panel) also forms a magic square with both diagonals “Latin” in that each stamp appears once in the main forwards and backwards diagonals; the only other such type is $b_{342}$ (Figure 4.5, left panel). Issued by Portugal in 1997, the stamps shown in Figure 4.5 (right panel) depict the Pyrenean desman ($Galemys pyrenaicus$), a small semi-aquatic mammal that lives in the Iberian peninsula; its body is like that of a muskrat, its nose like that of a hedgehog and its feet like that of a duck-billed platypus. The desman uses its snout as a periscope to breathe above the water’s surface.

These two PLS (Figure 4.5, the two panels), respectively of type $b_{342}$ and $b_{423}$, are orthogonal to each other in that the pair defines a philatelic

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4 Listed on the WWF Groth AG website [13] but apparently not listed by Scott [26].
“Graeco–Latin square” from which, using “Euler’s algorithm” Dénes & Keedwell [9, p. 212], we find
\[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
3 & 4 & 1 & 2 \\
4 & 3 & 2 & 1 \\
2 & 1 & 4 & 3
\end{pmatrix} + 4 \begin{pmatrix}
1 & 2 & 3 & 4 \\
4 & 3 & 2 & 1 \\
2 & 1 & 4 & 3 \\
3 & 4 & 1 & 2
\end{pmatrix} - 4 \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1
\end{pmatrix} = \begin{pmatrix}
1 & 6 & 11 & 16 \\
15 & 12 & 5 & 2 \\
8 & 3 & 14 & 9 \\
10 & 13 & 4 & 7
\end{pmatrix}
\]
which is a fully-magic matrix in “classic” form in that the numbers 1, 2, . . . , 16 each appear once.

![Figure 4.6: Guinea 2000, Groth 275, PLS type c423.](image)

The sheetlet in Figure 4.6 was issued by the Republic of Guinea (RÉPUBLIQUE de Guinée). Depicted is the Guinea-baboon (Papio papio) in row 1, columns 1 and 2, and the collared mangabey (Cercocebus torquatus) in row 1, columns 3 and 4. The Guinea-baboon is a baboon from the Old World monkey family with reddish brown hair and a hairless, dark-violet or black face. The collared mangabey is a long-tailed and long-legged, relatively large monkey with conspicuous white eyelids and short, backwardly-directed hairs on the crown. This PLS is of type c423 and the only such PLS that we have identified.

The sheetlet shown in Figure 4.7 (left panel) was issued by the Republic of The Gambia, commonly known as Gambia, the smallest country on the African continental mainland. It is bordered to the north, east, and south by Senegal, and has a small coast on the Atlantic Ocean in the west. Depicted is the Black Crowned Crane (Balearica pavonina), a bird in the family Gruidae. This species and the closely related Grey Crowned Crane (Balearica regulorum), which prefers wetter habitats for foraging, are the only cranes that can nest in trees. This PLS is of type b432, which is in Sudoku form and is block-Latin.
Our 10th and last example of a PLS (Figure 4.7, right panel) is of type $d324$ and the only PLS of this type that we have found; we call this type *criss-cross* since the same stamp appears in each cell in the main forwards diagonal, and the same stamp appears in each cell in the main backwards diagonal. The stamps were issued by the Pitcairn Islands, well known as the home of the descendants of the *Bounty* mutineers and the accompanying Tahitians. In the top row, left to right, are the black noddy (*Anous minutus*), the brown noddy (*Anous stolidus*), the black-grey ternlet (*Procelsterna cerulea*) and the sooty tern (*Sterna fuscata*).

4.1. $4 \times 4$ Rainbow Latin squares. The sheetlet of 16 stamps from Pakistan (Figure 4.8) is almost a PLS—even though 4 different water birds each appear 4 times, the 4 copies of the stamp for each bird are not all identical. We may think of this as a one-step backwards circulant PLS with a superimposed “rainbow-square” with each row defining (the 4 parts of) a rainbow. We will call this a rainbow-PLS (R-PLS), more precisely a row-rainbow-PLS (RR-PLS). Depicted are 4 kinds of water birds: (a) gadwall, (b) common shelduck, (c) mallard, and (d) greylag goose. This is the only RR-RLS that we have found.

We have, however, found a column-rainbow-PLS (CR-PLS). Issued by Nepal in 2005, this PLS (Figure 4.9) is of the very rare fully-magic PLS-type $b342$ with the 4 stamps in each column marked by a different colour. Depicted are 4 kinds of mammals: (a) gangetic dolphin, (b) Indian pangolin, (c) Asiatic white elephant, and (d) clouded leopard.
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Figure 3.1: Pakistan 1992, Scott 790a–790d.

Figure 4.8: Pakistan 1992, Scott 790ad.

Figure 4.9: Nepal 2005, Scott 762ad.
5. PLS with $n = 5$

We now look at PLS with $n = 5$ for which we have identified just 17 examples (including 8 embedded in two panes each with 100 stamps) with just 4 different patterns. We have identified $5 \times 5$ PLS from 6 “countries” (stamp-issuing authorities): Canada (8), Pakistan (1), Swaziland (1), Transkei (5), Turkey (1), and the USA (1). These 17 PLS are all of the circulant type, either one- or two-step backwards (C1B, C2B) or forwards (C1F, C2F).

A $5 \times 5$ Latin square is said to be in standard-form when the first row is 1, 2, 3, 4, 5 and in reduced-form when it is in standard form and in addition the first column is 1, 2, 3, 4, 5. From the well-known Statistical Tables by Fisher and Yates [12, Table XV, p. 86], we find that there are 56 reduced-form $5 \times 5$ Latin squares (in contrast to just 4 reduced-form $4 \times 4$ Latin squares and just 1 reduced-form $3 \times 3$ Latin square) yielding $24 \times 56 = 1344$ possible distinct types of $5 \times 5$ PLS in standard-form. Of these 1344 types we have identified PLS of just these 4 types: C1B, C1F, C2B, C2F:

\[
C1B = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
2 & 3 & 4 & 5 & 1 \\
3 & 4 & 5 & 1 & 2 \\
4 & 5 & 1 & 2 & 3 \\
5 & 1 & 2 & 3 & 4 \\
\end{pmatrix}, \quad C1F = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
5 & 1 & 2 & 3 & 4 \\
4 & 5 & 1 & 2 & 3 \\
3 & 4 & 5 & 1 & 2 \\
2 & 3 & 4 & 5 & 1 \\
\end{pmatrix},
\]

\[
C2B = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
3 & 4 & 5 & 1 & 2 \\
5 & 1 & 2 & 3 & 4 \\
2 & 3 & 4 & 5 & 1 \\
4 & 5 & 1 & 2 & 3 \\
\end{pmatrix}, \quad C2F = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
4 & 5 & 1 & 2 & 3 \\
2 & 3 & 4 & 5 & 1 \\
3 & 4 & 5 & 1 & 2 \\
5 & 1 & 2 & 3 & 4 \\
\end{pmatrix}.
\]

5.1. **One-step circulants: C1B, C1F.** We have identified 5 PLS of the one-step backwards circulant type C1B: 1 from Swaziland and 4 from Canada and 10 of the one-step forwards circulant type C1F: 5 from Transkei, 1 from Turkey and 4 from Canada. The 8 PLS from Canada are embedded in two panes each of size $10 \times 10$.

A $5 \times 5$ PLS of type C1B was issued by Swaziland in 1982 for “Wildlife Conservation”, the first of a series of 4 sets of five stamps, see Figure 5.1. This first set was one of the earliest sets of stamps issued for the WWF.

Depicted in Figure 5.1 is Pel’s fishing-owl (*Scotopelia peli*), a large species of owl which feeds nocturnally on fish snatched from the surface of lakes and rivers. Printed on the stamps are the words *Sikhova setinhlanti*, which is fishing-owl in SiSwati, a Bantu language spoken in Swaziland and South Africa. Depicted are (top row left to right) male, female, pair, nest and egg,
adult (with youngster). The Kingdom of Swaziland is a landlocked country in southern Africa, bordered by South Africa on three sides except to the east, where it borders Mozambique.

Pel’s fishing owl is named after Hendrik Severinus Pel (1818–1876), who was a member of the technical staff of the Rijksmuseum van Natuurlijke Historie in Leiden from 1832–1840 and then worked for the Dutch government of the Gold Coast with the understanding that in his spare time he would collect natural history specimens for the Leiden Museum. In the mid-19th century, The Netherlands occupied six trade posts in what is now Ghana.
These stamps from Swaziland (Figure 5.1) seem to be quite rare. The 2010 *Scott Standard Postage Stamp Catalogue* lists a single stamp, unused, at US$16.00 and a strip of five stamps, unused, at US$125.00, while on the Steurs WWF website the full PLS is for sale at US$1609 on October 31, 2010 (US$1655.20 on January 29, 2010, up from US$1483.10 on June 12, 2008). Delcampe [8] has a strip of 5 stamps (row 2) for sale at 62 Euros (US$86.46) on October 31, 2010, while the Groth WWF website lists these stamps as a “forerunner” (Groth #F40) of those issued for the WWF and lists a strip of 5 stamps for sale 150 Swiss francs (US$152.28 but not in stock).

![Figure 5.2. Turkey 1960, Scott 1488–1492, PLS type C1F.](image)

A 5 × 5 PLS of type CF1 was issued in 1960 by Turkey (shown in Figure 5.2), the earliest 5 × 5 PLS that we have identified (the earliest 2 × 2 PLS that we have found was issued by Canada in 1960). This PLS was issued for the 1960 Summer Olympics held in Rome, Italy, August 25–September 11, 1960. Rome had been awarded the organization of the 1908 Summer Olympics, but after the 1906 eruption of Mount Vesuvius, was forced to decline and pass the honors to London. At the 1960 Olympics, Turkey won 7 gold medals and 2 silver—all 9 for wrestling. Depicted on the stamps are (top row, left to right): hurdling, soccer, steeplechase, basketball, wrestling.
5.2. **5×5-embedded PLS.**

![Figure 5.3. Canada 1970, Scott 519–523.](image)

Four 5 × 5 PLS are embedded in each of two panes of 100 stamps, one pane with 5-cent stamps (Figure 5.3), the other with 6-cent stamps. These panes of stamps were issued by Canada for Christmas 1970 featuring children’s designs. The pattern in each of the two panes is the same:

\[
\begin{pmatrix}
A & AF \\
FA & FAF
\end{pmatrix} = \begin{pmatrix}
I \\
F
\end{pmatrix} A \begin{pmatrix}
I & F
\end{pmatrix}, \quad F = \begin{pmatrix}
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0
\end{pmatrix}.
\]

Here the 5 × 5 matrix \( A \) denotes the one-step backwards circulant matrix \( C_1B \), the matrix \( I \) the 5 × 5 identity matrix, and \( F \) the 5 × 5 flip matrix. The PLS defined by \( AF \) (top right corner) and \( FA \) (bottom left corner) are one-step forwards circulants of type \( C_1F \) and \( FAF \) (bottom right corner) is a one-step backwards circulant of type \( C_1B \) like the PLS defined by \( A \) (top left corner), but with the stamps arranged differently. There are, therefore, eight distinct PLS in all, with two one-step forwards circulants and two one-step backwards circulants in each of the two panes. An interesting feature of these stamps is that just one of the five stamps appears four times contiguously in a block of four and so only in the centre of the full pane (Figure 5.3).
Two-step circulants: C2B, C2F. The only $5 \times 5$ PLS of the two-step circulant types C2B and C2F that we have identified are from Pakistan (Figure 5.4) and the USA (Figure 5.5). These PLS have several interesting properties. In particular they are both magic-Latin in that each of the 5 stamps appears once in the two main diagonals. Moreover, [23]: “Of Latin squares used for crop experiments with a single set of treatments, the earliest examples (published in 1924 [36]) are $5 \times 5$ squares of the types [C2B and C2F] known as Knut Vik or “knight’s move” designs; these are squares where all cells containing any one of the treatments can be visited by a succession of knight’s moves (as in chess) and where no two diagonally adjacent cells have the same treatment.”

As noted by Fisher in The Design of Experiments [11, p. 78]: “In this arrangement the areas bearing each treatment are nicely distributed over the experimental area, so as to exclude all probability that the more important components of heterogeneity should influence the comparison between treatments.” In 1931 Tedin [33] and in 1951 Nissen [21] observed that there are just two possible arrangements of the $5 \times 5$ Knut Vik design in standard-form: C2B and C2F. These designs are pandiagonal, in that all diagonals (the two main diagonals and all broken diagonals) contain each of the five treatments precisely once.
A PLS of type C2B was issued by Pakistan (Figure 5.4) for the National Philatelic Exhibition in Lahore on Universal Postal Union Day (October 9, 2004). This PLS depicts five kinds of tropical fish (top row, left to right): striped gourami (*Colisa fasciata*), black widow tetra (*Gymnocyprinus ternetzi*), yellow dwarf cichlid (*Apistogramma borellii*), tiger barb (*Carpofa tetrazona*), neon tetra (*Paracheirodon innesi*).

A PLS of type C2F was issued by the USA (Figure 5.5) depicting five antique automobiles from the late 19th and (very) early 20th century. In the top row (left to right): 1893 Duryea, 1898 Columbia, 1901 White, 1894 Haynes, 1899 Winton.

6. Acknowledgements

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