MATH 340: Discrete Structures II. Winter 2017.
Due in class on Wednesday, February 15th.

Assignment #2: Planar graphs.

1. *Euler's formula.*

a) Let G be a planar graph, such that every vertex of G has degree at least five, and at least one vertex of G has degree eight. Show that G has at least fifteen vertices.

b) Let G be a triangulation of the plane. Show that the number of faces of G is even.

2. *Coloring planar graphs.*

a) Show without using the Four Color Theorem that if a planar graph G has no K_3 subgraph then $\chi(G) \leq 4$.

b) Prove or disprove the following statement: If a planar graph G has no K_4 subgraph then $\chi(G) \leq 3$.

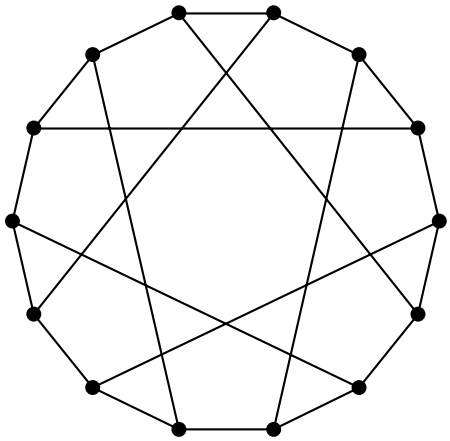
Hint: In a) show that G contains a vertex of degree at most three.

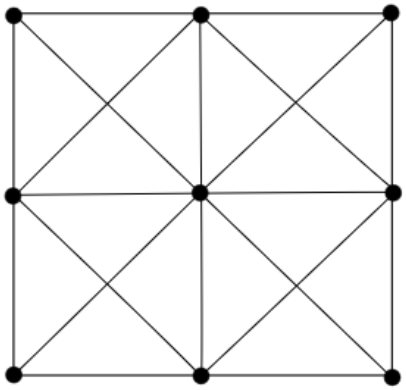
3. *Art Gallery theorem.*

a) Let P be a polygon in the plane such that at most two angles of P exceed $180°$. Show that P can be guarded by two guards.

b) Give an example of a polygon as in a) showing that two guards are sometimes required.

4. *Kuratowski's theorem.* Let G be a non-planar graph. Suppose further that $G \setminus e$ is planar for every edge e of G. Show that at most six vertices of G have degree three or greater.





5. *Testing planarity.* Determine whether the above two graphs are planar. (For each graph either provide a planar drawing, or prove that this graph is not planar.)