Name _____

**MATH 340: Discrete structures II. Winter 2016.**
Midterm Exam
Thursday, February 18th, 2016, 8:35-9:55

The questions have to be answered in the booklets provided.

Write your answers clearly. Justify all your answers. You may quote any result seen in the lectures without proving it.

Use of notes, textbooks, calculators, computers, cell-phones, etc. is not permitted.

Problem	Your score
1	
2	
3	
Total	
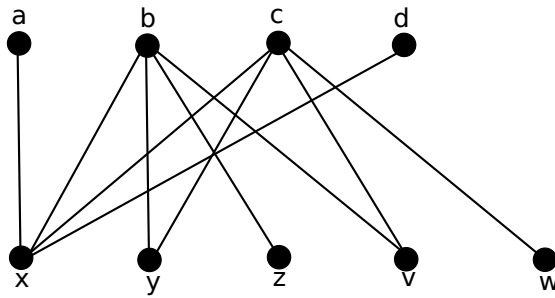
1. *Reminder:* $\tau(G)$ denotes the minimum size of a vertex cover of G, and $\nu(G)$ denotes the maximum size of a matching in G.

a) State Kőnig's theorem, relating these two parameters. [2 points]

b) Find $\nu(G)$ and $\tau(G)$ in the graph G on the figure above. [3 points]

c) Let G be a bipartite graph with m edges and maximum degree d. Show that $\nu(G) \geq m/d$. [5 points]
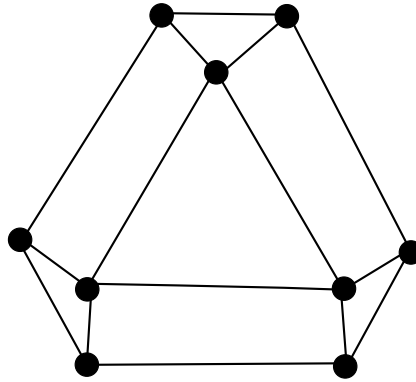
2.

a) Define the chromatic number $\chi(G)$ of a graph. [2 points]

b) Find $\chi(G)$ for the graph drawn on the figure above. [2 points]

c) Let G be a graph such that $E(G)$ can be partitioned into two sets E_1 and E_2 so that $G \setminus E_1$ and $G \setminus E_2$ are both planar. Show that $\chi(G) \leq 12$.
Hint: Use induction. In the induction step, using a bound on the number of edges in a planar graph, show that G has a vertex of degree at most 11.
[6 points]
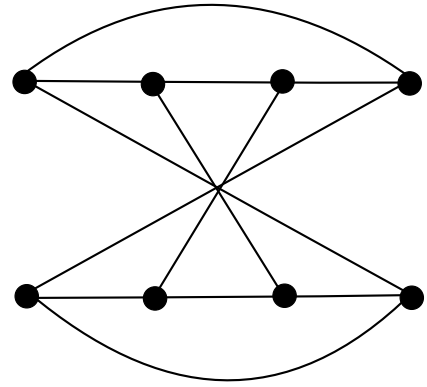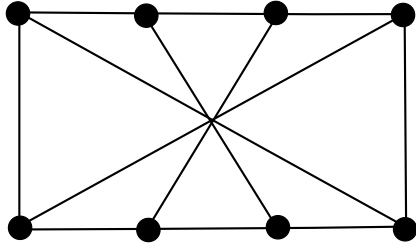
3.

a) Define a minor of a graph G. [2 points]

b) State Kuratowski's theorem. [2 points]

c) Explain whether or not each of the two graphs on the figure above is planar.
[6 points]