MATH 240: Discrete structures I. Fall 2011

Assignment #5: Combinatorics and Graph Theory.

Due Monday, November 28th.

1. *Fibonacci Numbers.*
Show that for every positive integer n the Fibonacci number F_{5n} is divisible by 5.

2. *Recurrence relations.*

(a) Solve the recurrence relation

$$p(n) = 4p(n-1) + 5$$

with $p(0) = 1, p(1) = 9$.

(b) Let f_n be the number of subsets of $\{1, 2, \ldots, n\}$ that contain no three consecutive integers. Find a recurrence for f_n.

3. *Inclusion-Exclusion.*

(a) An integer n is called *square free* if it does not have a divisor of the form k^2 where $k \in \{2, 3, \ldots, n\}$. Find the number of square-free integers between 1 and 120.

(b) In how many permutations of the set $\{0, 1, 2, \ldots, 9\}$ do either of 0 and 1, or 2 and 0, or 3 and 2 appear consecutively? (For example, we do not count

$$(5, 6, 0, 4, 9, 2, 3, 7, 8, 1),$$

as we want 3 and 2 to appear consecutively in that order. We count

$$(3, 5, 7, 2, 0, 1, 9, 8, 4, 6),$$

both 0 and 1, and 2 and 0 appear consecutively in it.)

4. *Counting integer solutions.*

(a) How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 30,$$

such that $3 \leq x_i \leq 10$ for every $1 \leq i \leq 4$?

(b) How many non-negative integer solutions are there to the inequality

$$x_1 + x_2 + \ldots + x_k \leq n \qquad ?$$

5. *Graph Degrees.*

(a) Does there exist a simple graph with 7 vertices and the following degrees: $\{0, 1, 2, 2, 2, 3, 6\}$?

(b) How many simple graphs are there with the vertex set $\{A, B, C, D\}$ such that two of the vertices have degree one and the remaining two vertices have degree two?