MATH 240: Discrete structures I. Fall 2011

Assignment #1: Sets and Logic. Due Friday, September 23rd.

1. *Venn Diagrams.* Draw Venn Diagrams for the following sets.

a) $(A - B) \cap C$,

b) $A \oplus (B \cup C)$.

2. *Set Identities.* Prove the following set identities.

a) $(A - B) \cap C = (A \cap C) - (B \cap C)$,

b) $(A \oplus B) \oplus (A \oplus C) = B \oplus C$.

3. *Propositions.* Which of the following sentences are propositions?

a) It rained yesterday.

b) The last digit of the smallest prime number larger than 100^{100} is 1.

c) This sentence is false.

d) $6 + 5 = 10$.

4. *Truth tables.* Use a truth table to verify the following equivalence

$$\neg(P \vee (Q \wedge (\neg R))) \leftrightarrow (\neg P) \wedge ((\neg Q) \vee R).$$

5. *Conditional equivalence.* Which of the following implications are true?

a) If $2 + 2 = 5$ then $2 + 2 = 6$.

b) If $2 + 2 = 4$ then the world is flat.

c) If both of the previous statements are true then $2 + 2 = 7$.

6. *Tautologies.* Which of the following are tautologies? If the statement is a tautology give a proof using the appropriate rules of logic demonstrated in class at each step of the proof. (Avoid using truth tables if possible.)
If not, then justify your answer by giving a counter-example, i.e., a truth assignment which results in a false value.

a) $p \rightarrow (p \vee q)$.

b) $((p \vee q \vee r) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$.

c) $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \rightarrow q) \rightarrow r)$.

7. *Circuits.* Suppose we have a committee of four people. Design a circuit which determines if exactly two of them vote yes on an issue.