Department of Mathematics
Final Examination

21 December 2015
09:00-12:00

CALCULUS II
201-NYB-05

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Student name: ____________________

Student number: ____________________

Instructor: ____________________

Instructions

1. Write all of your solutions in this booklet and show all supporting work.
2. If the space provided is not sufficient, continue the solution on the opposite page.
3. Check that this booklet contains 3 pages, excluding this cover page.
4. Remember that the use of a calculator is not permitted.

1. Evaluate the following integrals:

(a) $\displaystyle\int \frac{\tan^5(\ln x)\sec^3(\ln x)}{x}\,dx$

(b) $\displaystyle\int x\arcsin x\,dx$

(c) $\displaystyle\int \frac{x^2}{\sqrt{4x^2-9}}\,dx$

(d) $\displaystyle\int \frac{8x^2+4x+5}{(x+1)^2(2x-1)}\,dx$

(e) $\displaystyle\int_5^6 \frac{1}{\sqrt{-x^2+10x-21}}\,dx$

(f) $\displaystyle\int \sqrt{x}e^{\sqrt{x}}\,dx$

(g) $\displaystyle\int_1^{\sqrt{3}} \frac{1}{x^2\arctan x+\arctan x}\,dx$

2. Given that $f(-2)=-3$; $f'(-2)=5$; $f(1)=3$; $f'(1)=2$
Evaluate $\displaystyle\int_{-2}^{1} xf''(x)\,dx$

3. Evaluate the following limits:

(a) $\displaystyle\lim_{x\to\pi}\frac{\sin^2 3x}{1+\cos x}$

(b) $\displaystyle\lim_{x\to\infty} x(e^{3/x}-1)$

(c) $\displaystyle\lim_{x\to 0^+}(\cos x)^{1/x^2}$

4. Determine whether the following integrals converge or diverge:

(a) $\displaystyle\int_1^{\infty}\frac{1-\ln x}{x^2}\,dx$

(b) $\displaystyle\int_0^3\frac{dx}{x^2-2x+1}$

5. Solve the differential equation:
$\cos^2 x\dfrac{dy}{dx}=e^{-y}\sin x$; $y(0)=0$

6. Sketch the region $\mathcal{R}$ bounded by $y=x^2+1$ and $y=2x+4$ and find its area.

7. Let $\mathcal{R}$ be the region bounded by the functions $y = 1 + \cos x$, $y = 1$ and $0 \leq x \leq \frac{\pi}{2}$. Set up (**but do not evaluate**) the integrals to find the volume of the solid of revolution obtained by revolving $\mathcal{R}$ about;

(a) the $y-$axis

(b) the $x-$axis

(c) the line $x = 3$

8. Consider the sequence $a_n = n \sin\left(\frac{1}{n}\right)$

(a) Is the sequence $\{a_n\}$ convergent? If so, find it's convergence value or explain why it diverges.

(b) Is the series $\sum_{n=1}^{\infty} a_n$ convergent? Justify your answer.

9. Let $S_n = \frac{n}{n+2}$ be the sequence of partial sums for the series $\sum_{n=1}^{\infty} a_n$.

(a) Evaluate $\sum_{n=1}^{\infty} a_n$

(b) Find a_n

10. Determine whether the following series converge or diverge. Justify your answer by displaying a proper solution.

(a) $\sum_{n=1}^{\infty} \left(\frac{2n}{3n+2} - \frac{1}{n\sqrt{n}} \right)$

(b) $\sum_{n=1}^{\infty} \frac{1}{n\,7^n}$

(c) $\sum_{n=1}^{\infty} \frac{5\sqrt{n}}{(n+1)^2}$

11. Determine whether the following series are absoutely convergent, conditionally convergent or divergent. Justify your answer by displaying a proper solution.

(a) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\ln(2n)}$

(b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2n+1)!}{n^2\,7^n}$

12. Find the radius and interval of convergence for the power series,
$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^{2n}(x-1)^n}{3n+1}$

13. Find the Maclaurin series expansion for $f(x) = \dfrac{x}{x+1}$

Answers Fall 2015

1. (a) $\frac{1}{7}\sec^7(\ln x) - \frac{2}{5}\sec^5(\ln x) + \frac{1}{3}\sec^3(\ln x) + \text{c}$

(b) $\frac{x^2}{2}\arcsin x - \frac{1}{4}\arcsin x + \frac{1}{4}x\sqrt{1-x^2} + \text{c}$

(c) $\frac{9}{16}\left(\frac{2x}{3}\cdot\frac{\sqrt{4x^2-9}}{3} + \ln\left|\frac{2x}{3} + \frac{\sqrt{4x^2-9}}{3}\right|\right) + \text{c}$

(d) $2\ln|x+1| + \frac{3}{x+1} + 2\ln|2x-1| + \text{c}$

(e) $\pi/6$

(f) $2e^{\sqrt{x}}(x - 2\sqrt{x} + 2) + \text{c}$

(g) $\ln(4/3)$

2. 6

3. (a) 18

(b) 3

(c) $1/\sqrt{e}$

4. (a) Convergent

(b) Divergent

5. $y = \ln|\sec x|$

6. 32/3

7. (a) $\text{v} = 2\pi\int_0^{\pi/2} x\cos x\,dx$

(b) $\text{v} = \pi\int_0^{\pi/2} ((1+\cos x)^2 - (1)^2)\,dx$

(c) $\text{v} = 2\pi\int_0^{\pi/2} (3-x)\cos x\,dx$

8. (a) $\{a_n\}$ is convergent

(b) $\sum_{n=1}^{\infty} a_n$ is divergent

9. (a) 1

(b) $a_n = \frac{2}{(n+1)(n+2)}$

10. (a) Divergent by test for divergence

(b) Convergent by comparison test

(c) Convergent by limit comparison test

11. (a) Conditionally Convergent

(b) Divergent by Ratio Test

12. Radius = 1/9 ; Interval $8/9 < x \leq 10/9$

13. $\sum_{n=1}^{\infty}(-1)^{n+1}x^n$