1. (5 points) Given the graph of f below evaluate the following expressions. If appropriate use ∞, $-\infty$, or "does not exist".

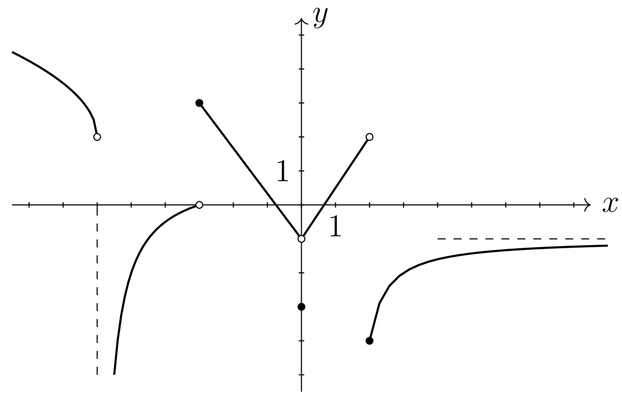
(a) $\lim_{x\to 0} f(x)$

(b) $\lim_{x\to\infty} f(x)$

(c) $\lim_{x\to -3^-} f(x)$

(d) $\lim_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$

(e) $\lim_{x\to -6^-} f(f(x))$



2. (10 points) For each of the following, evaluate the limit or show the limit does not exist.

(a) $\lim_{x\to 1} \dfrac{2x^2+x-3}{x^4-1}$

(b) $\lim_{t\to 2} \dfrac{2t+1-\sqrt{8t+9}}{t-2}$

(c) $\lim_{x\to 4} \dfrac{x^2-4x}{x^2-8x+16}$

(d) $\lim_{\theta\to 0} \dfrac{\sin^2(2\theta)}{\theta\tan(3\theta)}$

(e) $\lim_{x\to\infty} (2x-\sqrt{4x^2-3x})$

3. (4 points) Use the definition of continuity to determine the constants A and B that will make $f(x)$ continuous at $x=-1$ and 0.

$$f(x) = \begin{cases} \dfrac{\frac{1}{x-1}-\frac{1}{x}}{\frac{1}{x}-\frac{1}{x+1}} & \text{if } x \notin \{-1,0,1\}, \\ A & \text{if } x=-1, \\ B & \text{if } x=0, \\ 5 & \text{if } x=1 \end{cases}$$

4. (18 points) Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answers.

(a) $y = \dfrac{\sqrt[3]{x}-3x^4+7}{2\sqrt{x}}$

(b) $y = \sin^3(x) - \ln(\csc x) - \pi^{3x}$

(c) $y = \dfrac{\cos(7x) - \sqrt{5x}}{\log_3(x)}$

(d) $y = \dfrac{4x^6 \cot^4(3x)}{\sqrt{5^x \ln x}}$ (Use Logarithmic Differentiation)

(e) $y^2 \tan(x + y) = 4$

(f) $y = (3x + 2)^{xe^{2x}}$

5. (4 points) Find the equation of the line tangent to the curve $(y - 5)^5 = x^2 + 2xy - 34$ at $(3, 4)$.

6. (4 points) Let $g(1) = 2$, $g'(1) = -3$, $g'(2) = -1$, and $g''(2) = -4$.

If $f(x) = \dfrac{4xg(x^2)}{g'(2x)}$, determine

(a) $f'(x)$

(b) $f'(1)$

7. (4 points) Find the absolute extrema of $y = x - \sqrt{x}$ over the interval $[0,\ 4]$.

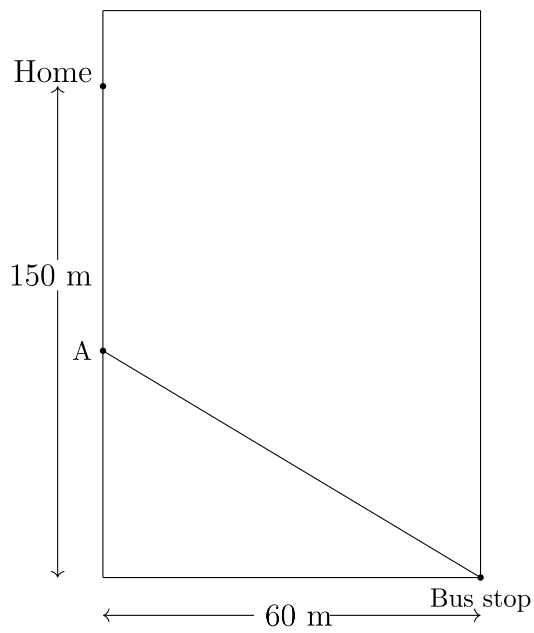
8. (5 points) Stephanie is sitting on the ground 10 feet from the spot where a hot air balloon is about to land. She is watching the balloon as it travels at a steady rate of 20 feet per second towards the ground. If θ is the angle between the ground and her line of sight to the balloon, at what rate is this angle changing at the instant the balloon hits the ground?

9. (10 points) Given

$$f(x) = \ln|x^2 - 9|\ ,\ f'(x) = \frac{2x}{x^2 - 9} \quad \text{and } f''(x) = \frac{-2x^2 - 9}{(x^2 - 9)^2},$$

Determine (as appropriate) the domain, any intercepts, any vertical and horizontal asymptotes, the intervals of increase and decrease, any local extrema, the intervals of concavity, and any inflection points. On the next page, sketch the graph of f.

10. (5 points) Stephen lives next to a park. He takes a bus from work which drops him off on the opposite corner of the park (see below). Normally, he saves time by cutting diagonally through the park instead of going along the sidewalk. In the winter, however, the park is covered in snow, so he will cut across to a point A as pictured, and continue on the sidewalk. He's able to walk $\sqrt{5}$ m/s on the sidewalk, but only 1 m/s through snow. What path should he take in order to minimize his walking time? (Time=Distance/Speed)



11. (12 points) Evaluate the following integrals.

(a) $\displaystyle\int \left(3e^x + x^3 + x^e + 3\right)\, dx$

(b) $\displaystyle\int_0^{\pi/4} \frac{\sin x + 1}{\cos^2 x}\, dx$

(c) $\displaystyle\int_1^4 \frac{(x+3)(x-2)}{\sqrt{x^3}}\, dx$

(d) $\displaystyle\int \left(\sin^3 x + \sin x \cos^2 x\right)\, dx$

12. (4 points) The acceleration (in m/s^2) of a particle moving in a straight line is given by the formula $a(t) = 2t + 1$. The initial velocity of the particle (at time $t = 0$) is -2 m/s.

(a) Find the velocity at time t.

(b) Find the total distance travelled during the time $0 \le t \le 3$.

13. (8 points)

(a) Estimate $\displaystyle\int_{-1}^{5} \left(\frac{x^2}{2} - 1\right) dx$ as a Riemann sum with $n = 3$, taking midpoints as sample points.

(b) Evaluate $\displaystyle\int_{-1}^{5} \left(\frac{x^2}{2} - 1\right) dx$ as a limit of Riemann sums. You may find the following summation formulas helpful: $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\qquad$ $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

14. (4 points) Given $F(x) = \int_x^{3x^2} \sqrt{1+\sin t}\,dt$, find $F'(x)$.

15. (3 points) It is given that f' is continuous on $\mathbb{R}$, and that

$$f(-1) = 7, \qquad f(3) = 1 \qquad \text{and} \qquad f(9) = 9.$$

Show that the equation $f'(x) - 1 = 0$ has at least one real solution.

ANSWERS

1. (a) -1

(b) -1

(c) 0

(d) $\frac{3}{2}$

(e) -4

2. (a) $\frac{5}{4}$

(b) $\frac{6}{5}$

(c) D.N.E.

(d) $\frac{4}{3}$

(e) $\frac{3}{4}$

3. $A = 0, B = -1$

4. (a) $\frac{dy}{dx} = \frac{-1}{12}x^{-7/6} - \frac{21}{4}x^{5/2} - \frac{7}{4}x^{-3/2}$

(b) $\frac{dy}{dx} = 3\sin^2(x)\cos(x) + \cot(x) - 3\ln(\pi)\pi^{3x}$

(c) $\frac{dy}{dx} = \frac{\left(-7\sin(7x) - \frac{5}{2\sqrt{5x}}\right)\log_3(x) - \frac{1}{x\ln 3}\left(\cos(7x) - \sqrt{5x}\right)}{(\log_3 x)^2}$

(d) $\frac{dy}{dx} = \frac{4x^6\cot^4(3x)}{\sqrt{5^x}\ln x}\left(\frac{6}{x} - \frac{12\csc^2(3x)}{\cot(3x)} - \frac{\ln 5}{2} - \frac{1}{2x\ln x}\right)$

(e) $\frac{dy}{dx} = \frac{-y^2\sec^2(x+y)}{2y\tan(x+y) + y^2\sec^2(x+y)}$

(f) $\frac{dy}{dx} = (3x+2)^{xe^{2x}}\left(e^{2x}\ln(3x+2) + 2xe^{2x}\ln(3x+2) + \frac{3xe^{2x}}{3x+2}\right)$

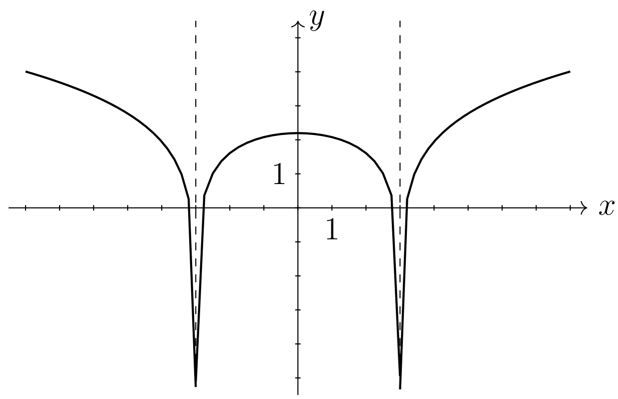
5. $y = -14x + 46$

6. 80

7. Absolute Maximum: 2
Absolute Minimum: $-\frac{1}{4}$

8. Decreasing at rate 2 rad/sec

9. Domain: $x \in \mathbb{R}\backslash\{-3, 3\}$
$y-$intercept: $(0, \ln 9)$
$x-$intercept: $(\pm\sqrt{8}, 0), (\pm\sqrt{10}, 0)$
Vertical asymptotes: $x = -3, x = 3$
Horizontal asymptotes: None
Increasing: $(-3, 0)$ and $(3, \infty)$
Decreasing: $(-\infty, -3)$ and $(0, 3)$
Local Maximum: $(0, \ln 9)$
Local Minimum: None
Concave up: Never
Concave down: $x \in \mathbb{R}\backslash\{-3, 3\}$
Inflection Points: None



10. He should walk to 120m from his home.

11. (a) $3e^x + \frac{x^4}{4} + \frac{x^{e+1}}{e+1} + 3x + C$
(b) $\sqrt{2}$
(c) $\frac{2}{3}$
(d) $-\cos(x) + C$

12. (a) $v(t) = t^2 + t - 2$
(b) $\frac{41}{6}$m

13. (a) 14
(b) 15

14. $F'(x) = -\sqrt{1 + \sin(x)} + 6x\sqrt{1 + \sin(3x^2)}$

15. By the MVT, there exists c_1 such that $f'(c_1) = \frac{1-7}{3-(-1)} = \frac{-3}{2}$ and c_2 such that $f'(c_2) = \frac{9-1}{9-3} = \frac{4}{3}$.

Since f' is continuous on $\mathbb{R}$, by the IVT, there exists c such that $f'(c) = 1$