

1. (a) Elliptic paraboloid ( $y \geq 0$ )  
 (b) Hyperboloid of one sheet  
 (c) The plane  $x + y + z = 2$
2. (a)  $x = t, y = \sqrt{8} + \sqrt{2}t, z = 1 + t$   
 (b)  $v = 2 \left( \frac{1}{t} + t \right)$   
 (c)  $\kappa(1) = \frac{\sqrt{2}}{8}$   
 (d)  $a_T(1) = 0$  and  $a_N(1) = 2\sqrt{2}$
3. (a) The limit does not exist.  
 (b) The limit is 2.
4. (a)  $dz = \frac{x}{\sqrt{x^2+y^2}}dx + \frac{y}{\sqrt{x^2+y^2}}dy$   
 (b)  $f(3.06, 3.92) \simeq f(3, 4) + dz|_{(3,4)} = 4.972$
5. (a)  $D_{\mathbf{u}}F(P) = 2\sqrt{3}$   
 (b)  $\|\nabla F(P)\| = \sqrt{206}$   
 (c) In the direction  $\frac{1}{\sqrt{206}}\langle 9, 10, 5 \rangle$   
 (d)  $9x + 10y + 5z = 33$   
 (e)  $D_{\vec{PQ}}F(P) = 0$   
 (f)  $\frac{\partial z}{\partial y} = -\frac{2x^2 + 2yz^3}{x + 3y^2z^2}$
6. Note that  $\frac{\partial z}{\partial x} = 2xy \frac{\partial f}{\partial u}$  and  $\frac{\partial z}{\partial y} = f(u) - 2y^2 \frac{\partial f}{\partial u}$  leading to the result.
7. The points  $(-1, -1)$  and  $(1, 1)$  are both local minima while  $(0, 0)$  is a saddle point.
8.  $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}})$
9. (a)  $\frac{1}{4}(1 - \cos(16))$   
 (b)  $\frac{\pi}{4}(\sin 5 - \sin 4)$   
 (c)  $\frac{81\sqrt{18}(2 - \sqrt{2})\pi}{5}$
10. (a)  $\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{\sqrt{1+x^2+y^2}}^{\sqrt{19-x^2-y^2}} dz dy dx$   
 (b)  $\int_0^{2\pi} \int_0^3 \int_{\sqrt{1+r^2}}^{\sqrt{19-r^2}} r dz dr d\theta$
11.  $\frac{16}{5}$
12.  $f^{(27)}(-6) = \frac{9}{28}$

$$13. \quad (a) \quad \frac{x^3}{5+x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{5^{n+1}} \quad R = \sqrt{5}$$

$$(b) \quad \frac{\arctan(3x^2)}{x} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{4n+1}}{2n+1} \quad R = \frac{1}{\sqrt{3}}$$

$$14. \quad \int_0^{0.1} x e^{-x^3} dx = \sum_{n=0}^{\infty} \frac{(-1)^n (0.1)^{3n+2}}{(3n+2)n!} \simeq 0.004998$$

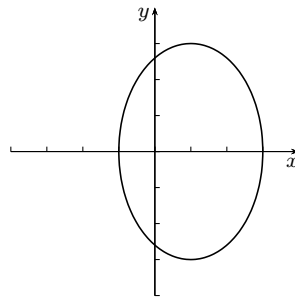
$$15. \quad (a) \quad f(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{(-1)^n (2n-1)!! (x-9)^n}{3^{2n+1} 2^n n!} \quad R = 9$$

$$(b) \quad R_2(x) = \frac{-15(x-9)^3}{8(3!)(z^{7/2})} \text{ which implies that } |R_2(9.5)| \leq 1.78 \times 10^{-5}$$

$$16. \quad (a) \quad \frac{dy}{dx} = \frac{-3 \cot t}{2} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{-3}{4 \sin^3 t}$$

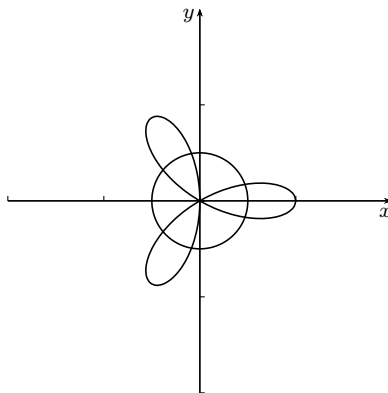
(b) Vertical tangents at  $(-1, 0)$  and  $(3, 0)$ ; horizontal tangents at  $(1, -3)$  and  $(1, 3)$

(c) The curve is the ellipse  $\frac{(x-1)^2}{4} + \frac{y^2}{9} = 1$



$$(d) \quad I = -12 \int_0^{\pi} \sin^2 t \, dt \quad \text{and} \quad A = -I$$

17. (a) The curves are a rose with 3 petals and a circle respectively



$$(b) \quad A = 3 \left( \int_0^{\pi/9} \frac{1}{4} d\theta + \int_{\pi/9}^{\pi/6} \cos^2(3\theta) d\theta \right)$$

$$(c) \quad L = 6 \int_0^{\pi/6} \sqrt{1 + 8 \sin^2(3\theta)} d\theta$$