(Marks)

1. Evaluate the following integrals.

(5) (a) $\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{6}{\sqrt{1-x^2}}\,dx$

(5) (b) $\displaystyle\int e^{-3x}\cos(2x)\,dx$

(5) (c) $\displaystyle\int_{1}^{\sqrt{e}} \frac{1}{x(\ln x - 1)}\,dx$

(5) (d) $\displaystyle\int \frac{e^{3x}}{9+e^{2x}}\,dx$

(5) (e) $\displaystyle\int \sqrt{\tan x}\,\sec^4 x\,dx$

(5) (f) $\displaystyle\int \frac{x^2+3x+1}{(2x^2+1)(x-3)}\,dx$

(5) (g) $\displaystyle\int \frac{\sqrt{x^2-9}}{x^2}\,dx$

2. Evaluate the following limits.

(3) (a) $\displaystyle\lim_{x\to\pi} \frac{1+\cos x}{(x-\pi)^2}$

(3) (b) $\displaystyle\lim_{x\to\infty} \left(1+\frac{2}{x}+\frac{1}{x^2}\right)^x$

3. Evaluate the following improper integrals.

(4) (a) $\displaystyle\int_{4}^{\infty} \frac{2}{x^2-2x}\,dx$

(4) (b) $\displaystyle\int_{0}^{2} \frac{1}{\sqrt[3]{1-x}}\,dx$

(4) 4. Find the value a such that the line $x = a$ divides the region bounded by $y = e^x$ and the x-axis from $x = 0$ to $x = \ln 5$ into two regions of equal area.

(6) 5. Let $\mathcal{R}$ be the region bounded by the curves $f(x) = x + \dfrac{16}{x}$ and $g(x) = 10$. Set up, but **do not evaluate** the integral for the volume obtained by rotating the region $\mathcal{R}$ about the following:

(a) The x-axis.

(b) The line $x = 10$.

(4) 6. Find the length of the curve $y = \frac{2}{3}(x^2+1)^{3/2}$ from $x = 0$ to $x = 3$.

(4) 7. Solve the differential equation $y' = xe^{2x^2+y}$ with $y(0) = 0$. Express y as a function of x.

8. Determine whether each sequence $\{a_n\}$ converges or diverges. If a sequence converges, find what it converges to. Justify your answers.

(2) (a) $a_n = (-1)^n \dfrac{2^n}{2^n+n^2}$

(3) (b) $a_n = \dfrac{5^n}{n!}$

(Marks)

(2) 9. (a) If the sequence $\{a_n\}$ converges, does the series $\sum a_n$ converge? Briefly justify.

(b) If the series $\sum a_n$ converges, does the series $\{a_n\}$ converge? Briefly justify.

(3) 10. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty}\left[\left(1+\frac{1}{n}\right)^n-\left(1+\frac{1}{n+1}\right)^{n+1}\right]$.

11. Determine whether the following series are convergent or divergent.

(3) (a) $\displaystyle\sum_{n=1}^{\infty}\ln\left(\frac{2n^2+3}{n^2+5}\right)$

(3) (b) $\displaystyle\sum_{n=1}^{\infty}ne^{-n^2}$

(3) (c) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin(n)}{n\sqrt{n}}$

12. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent.

(3) (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(\frac{4n-1}{25n+5}\right)^{\frac{n}{2}}$

(3) (b) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{\arctan n}{\sqrt{n^2+1}}$

(4) 13. Find the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty}\frac{(n!)^2(2x-1)^n}{(2n)!}$

(4) 14. Find the Taylor series for $f(x)=\sqrt{x}$ centered at 1. What is its radius of convergence?

Answers

1.(a) π (b) $\frac{e^{-3x}}{13}(2\sin(2x)-3\cos(2x))+C$ (c) $-\ln 2$ (d) $e^x-3\arctan\left(\frac{e^x}{3}\right)+C$

(e) $\frac{2}{3}\tan^{3/2}x+\frac{2}{7}\tan^{7/2}x+C$ (f) $-\frac{1}{4}\ln(2x^2+1)+\ln|x-3|+C$ (g) $\ln\left|\frac{x}{3}+\frac{\sqrt{x^2-9}}{3}\right|-\frac{\sqrt{x^2-9}}{x}+C$

2.(a) $\frac{1}{2}$ (b) e^2 3.(a) $\ln 2$ (b) 0 4. $a=\ln 3$ 5.(a) $\pi\displaystyle\int_2^8 100-\left(x+\frac{16}{x}\right)^2\,dx$

(b) $2\pi\displaystyle\int_2^8(10-x)\left(10-x-\frac{16}{x}\right)\,dx$ 6. 21 7. $y=\ln\left(\frac{4}{5-e^{2x^2}}\right)$ 8.(a) D (b) C→0

9.(a) No (b)Yes 10. $2-e$ 11.(a) D (b) C (c) C 12.(a) AC (b) CC

13. $-\frac{3}{2}<x<\frac{5}{2}$ 14. $1+\frac{1}{2}(x-1)+\displaystyle\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1\cdot3\cdot5\cdots(2n-3)}{2^n n!}(x-1)^n$ with $R=1$