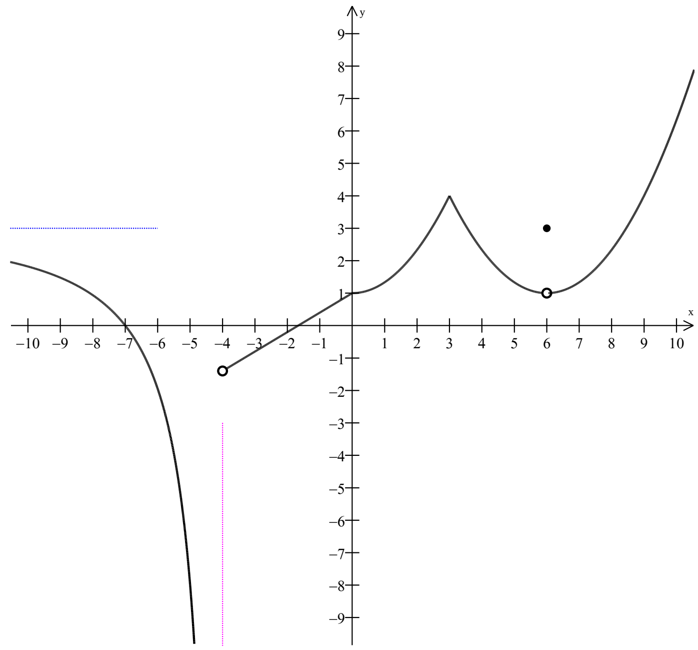
[Marks]

1. Use the graph of the function *f(x)* to answer each question. Where appropriate use $+\infty, -\infty$ or "does not exist". [3]

   a. $\lim\limits_{x \to -\infty} f(x) =$
   b. $\lim\limits_{x \to -4^-} f(x) =$
   c. $\lim\limits_{x \to -4} f(x) =$
   d. $\lim\limits_{x \to 6} f(x) =$
   e. $f(6) =$
   f. List the x-value(s) at which the *f(x)* is continuous but non differentiable



2. Evaluate the following. Where appropriate use $+\infty, -\infty$ or "does not exist".

   a. $\lim\limits_{x \to -2} \dfrac{6 - x - 2x^2}{x^2 - 4}$ [2]

   b. $\lim\limits_{x \to 6} \left(\dfrac{36 - x^2}{\sqrt{x+3} - \sqrt{x^2 - 27}} \right)$ [3]

   c. $\lim\limits_{x \to 0^-} \left(\dfrac{1}{x} - \dfrac{2}{x^2} \right)$ [2]

   d. $\lim\limits_{x \to -\infty} \left(\dfrac{x^2 + 5x - 6}{8x^3 - 27} - 4 \right) =$ [3]

3. Find all the x-values at which $f(x)$ is discontinuous, and determine the type of discontinuity at each value. [5]

$$f(x) = \begin{cases} \sqrt{3 - x} & \text{if} \quad x < -1 \\ \dfrac{|2x+1|}{2} & \text{if} \quad -1 \le x < 1 \\ \dfrac{-3x}{x^2 - x - 2} & \text{if} \quad x \ge 1 \end{cases}$$

4. Let *f(x)* be a continuous and differentiable function over [0,3] . If $f'(x) \le 2$ for all values of x and $f(0) = 4$. What is the maximum possible value of $f(3)$? [3]

5. Given the function $f(x) = \dfrac{2}{3-x}$, find $f'(x)$ using the LIMIT DEFINITION of the derivative. [4]

6. Find $\dfrac{dy}{dx}$ for each of the following:
   a. $y = 4x^3 - \dfrac{2}{\sqrt[5]{x^3}} - \dfrac{5^{(3x)}}{2} + \cot(x) - \log_2 x + \dfrac{e^3}{2}$ [3]
   b. $g(x) = \dfrac{\tan^2\left(e^x - 3\right)}{3x^2 + 5}$ [3]
   c. $y = \left(x^2 + 1\right)^{\csc(x)}$ [3]
   d. $y = \ln\left[\dfrac{\sqrt[4]{x^3 - 2x + 1}\,(6x-5)^2}{x^3(x^2 - 2x)^7}\right]$ [3]

7. The equation of motion of a particle is $s = t^3 - 3t^2$, where *s* is in meters and *t* is in seconds. [6]
   a. Find the velocity and acceleration as functions of t.
   b. When is the particle at rest? What is the acceleration at that moment?
   c. Find the velocity after 4 s.
   d. When is the acceleration zero?

8. Find the 25^{th} derivative of $f(x) = \sin(2x)$ [3]

9. For which values of x does the graph of $y = xe^{2x}$ have a horizontal tangent line ? [4]

10. Given the curve $x^3 + y^3 = 9xy$ (folium of Descartes) find the following:
    a. $\dfrac{dy}{dx}$ [2]
    b. The equation of the normal line to the curve at the point (2,4) [2]

11. 3 meters above the ground a fly is flying horizontally at a rate of 4 meters per minute. It passes over a small rock at noon. How fast is the distance between the fly and the rock increasing one minute later? [5]

12. Given $f(x) = \dfrac{-1}{x^2 - 1}$, $f'(x) = \dfrac{2x}{\left(x^2 - 1\right)^2}$, $f''(x) = -\dfrac{2(3x^2 + 1)}{\left(x^2 - 1\right)^3}$,find all : [10]
    a. The x and y intercepts.
    b. The vertical and horizontal asymptotes.
    c. The interval of increase and decrease.
    d. The local (relative) extrema(if any).
    e. The interval of upward and downward concavity.
    f. The inflection point(s) (if any).
    g. On the next page sketch the graph of *f(x)*. Label all intercepts, asymptotes, extrema and points of inflection.

13. Find the absolute extrema of $f(x)=\frac{x^3}{8}-\frac{3x}{2}$ on the closed interval $[-4\,,3]$. [4]

14. A cylinder with a closed top must have volume equal to $16\pi\ m^3$.What is the minimum amount of material (surface area) that can be used? [4]

15. Use differentiation to verify that the following formula is correct: [4]

   a. $\int \sec(x)\,dx=\ln|\sec(x)+\tan(x)|+c$

   b. Use the above formula to evaluate $\int_0^{\pi/4}\sec(x)\,dx$

16. Evaluate the following integrals.

   a. $\int_1^2 \frac{(2x-1)^2}{4x}\,dx$ [3]

   b. $\int \frac{x^2-9}{x-3}\,dx$ [3]

   c. $\int\left(\sqrt[5]{x}-\frac{5}{x}+5^x\right)dx$ [3]

17. Given $f''(x)=6x-2\cos(x)$; $f'(0)=-1$ and $f(0)=4$, find $f(x)$. [3]

18. a) Sketch and shade the region R bounded between the curve $y=1+\frac{4}{x}$, the x axis and the lines $x=1$ and $x=5$. [6]

   b) Find the exact value for the area of that region R
   c) Find an approximation for the area of R using a Riemann's sum with 4 equal subintervals and right endpoints.

19. Use the fundamental Theorem of Calculus to find $\frac{d}{dx}\int_x^7\sqrt{\tan\theta}\,d\theta$. [1]

Answers

1) a) 3 b) $-\infty$ c) Does Not Exist d) 1 e) 3 f) 0, 3

2) a) $-\frac{7}{4}$ b) $\frac{72}{11}$ c) $-\infty$ d) -4

3) Infinite discontinuity at x = 2 , and Jump Discontinuity at x = - 1

4) $f(3) \le 10$ 5) $\frac{2}{(3-x)^2}$ 6) a) $\frac{dy}{dx} = 12x^2 + \frac{6}{5x^{8/5}} - \frac{3}{2}5^{(3x)}\ln 5 - \csc^2 x - \frac{1}{x\ln 2}$

6) b) $g'(x) = \frac{2e^x(3x^2+5)\tan(e^x-3)\sec^2(e^x-3) - 6x\tan^2(e^x-3)}{(3x^2+5)^2}$

6)c) $y' = (x^2+1)^{\csc x}\left[-\csc x \cot x \ln(x^2+1) + \csc x \frac{2x}{x^2+1}\right]$

6) d) $y' = \frac{1}{4}\left(\frac{3x^2-2}{x^3-2x+1}\right) + \frac{12}{6x-5} - \frac{3}{x} - \frac{14(x-1)}{x^2-2x}$

7) a) $v(t) = 3t^2 - 6t$; $a(t) = 6t - 6$ b)at t= 0 s. or t= 2 s. $a(0) = -6m/s^2$; $a(t=2) = 6m/s^2$

c) $v(4) = 24m/s$ d) at t = 1 s.

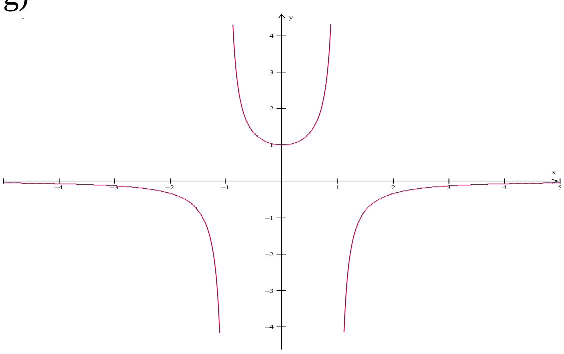
8) $f^{25th}(x) = 2^{25}\cos(2x)$ 9) $x = -\frac{1}{2}$ 10) a) $\frac{dy}{dx} = \frac{3y - x^2}{y^2 - 3x}$ b) $y = -\frac{5}{4}x + \frac{13}{2}$

11) $\frac{16}{5}m/\min$ 12)a) $(0,1)$, no x intercept b)vertical Asymptote at $x = \pm 1$; horizontal Asymptote at y = 0

c) *f(x)* increases on $(0,\infty)$, and decreases on $(-\infty,0)$ d) Local minimum at $(0,1)$

e) f(x) is concave down on $(-\infty,-1)\cup(1,\infty)$ and concave up on $(-1,1)$ f) no inflection point

g)



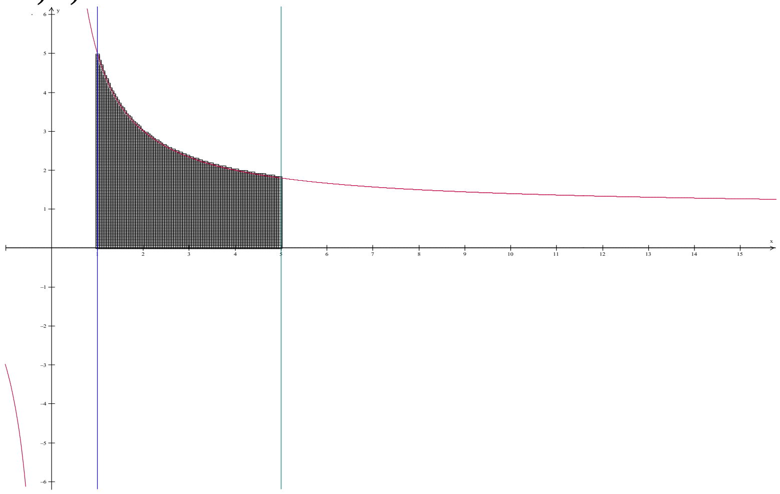
13) absolute max. $(-2,2)$, absolute min $(-4,-2)$ *and* $(2,-2)$ 14) $24\pi\ m^2$

15) a) $\frac{d}{dx}(\ln|\sec x + \tan x| + c) = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \sec x \frac{\tan x + \sec x}{\sec x + \tan x} = \sec x$

b) $\ln\left(1 + \frac{2}{\sqrt{2}}\right)$ 16) a) $\frac{1}{4}\ln 2 + \frac{1}{2}$ b) $\frac{x^2}{2} + 3x + c$ c) $\frac{5}{6}x^{6/5} - 5\ln|x| + \frac{5^x}{\ln 5} + c$

17) $f(x) = x^3 + 2\cos x - x + 2$

18) a)



b) $4(1+\ln 5)$ c) $\frac{137}{15}$

19) $-\sqrt{\tan x}$