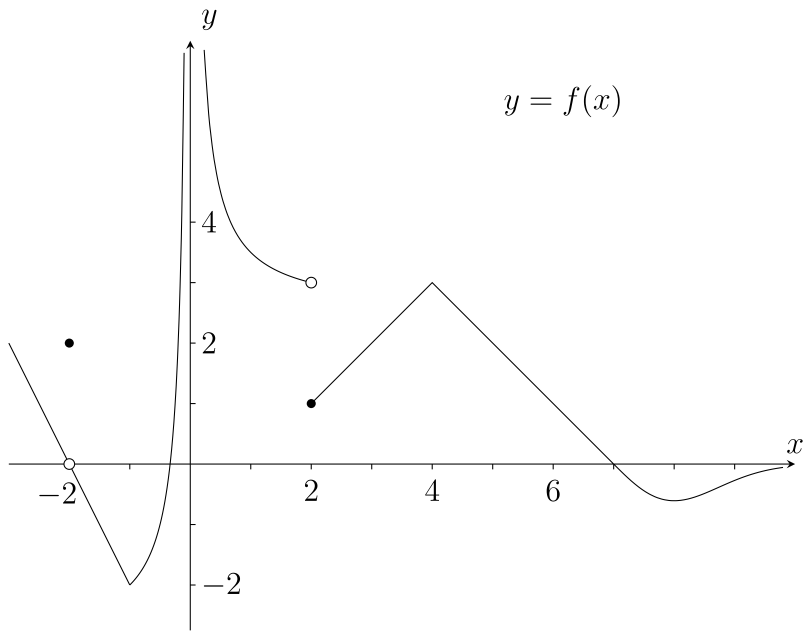
1. (6 points) Given the graph of f below, evaluate each of the following. Use ∞, $-\infty$ or "does not exist" where appropriate.



(a) $\lim_{x\to -2} f(x)$

(b) $\lim_{x\to 0} f(x)$

(c) $f'(4)$

(d) $\lim_{x\to \infty} f(x)$

(e) $\lim_{h\to 0} \dfrac{f(6+h)-f(6)}{h}$

(f) $\lim_{x\to 2}[f(x)-2]^2$

2. (10 points) Evaluate each of the following limits.

(a) $\lim_{x\to 5} \dfrac{50-2x^2}{2x^2-9x-5}$

(b) $\lim_{x\to -\infty} \dfrac{\sqrt{4x^6-9x}}{x^3}$

(c) $\lim_{x\to \infty} (e^x - e^{2x})$

(d) $\lim_{x\to 3^+} \dfrac{|6-2x|}{\sqrt{x-3}}$

(e) $\lim_{x\to 0} \dfrac{6x}{\sin 3x\, \cos 4x}$

3. (5 points) Let

$$f(x) = \begin{cases} \dfrac{x^2-4}{x^2-x-6} & \text{if } x \leqslant -1, \\ \frac{1}{4}x+1 & \text{if } -1 < x < 5, \text{ and} \\ \dfrac{1}{x^2-10x-24} & \text{if } x \geqslant 5. \end{cases}$$

Find the numbers at which f is not continuous. For each discontinuity that you find, specify whether the discontinuity is removable, jump or infinite.

4. (4 points) Use the limit definition of the derivative to find $f'(x)$, where $f(x) = \dfrac{1}{x^2+1}$.

5. (15 points) Find $\dfrac{dy}{dx}$ for each of the following.

(a) $y = 5^{\cot x} + \sec(4x^2) - 2e^{\pi+1}$

(b) $y = \tan^3(xe^x)$

(c) $y = \sqrt{\dfrac{x^3 \sin(2x)}{(x+1)^5}}$

(d) $e^{xy} - 3x^2 - 3y^2 = 2$

(e) $y = \left(\dfrac{2x-3}{\cos x}\right)^x$

6. Consider the curve defined by $xy^2 - x^3y = 6$.

(a) (2 points) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) (3 points) Find all points on the curve whose x-coordinate is 1, and write an equation of the tangent line at each of these points.

7. Consider the function defined by $f(x) = x^3 - 7x - 10$.

(a) (1 point) Use the Intermediate Value Theorem to show that a zero exists on the interval $[-1, 4]$.

(b) (2 points) Find the number in $(-1, 4)$ that satisfies the conclusion of the Mean Value Theorem.

(c) (1 point) Use Rolle's Theorem to show that there is a number c in $(-1, 3)$ such that $f'(c) = 0$.

8. (5 points) A conical tank, with its vertex down, has a diameter of 8 m and a depth of 16 m. Water flows into the tank at a rate of 5 m^3 per minute. Find the rate at which the water is rising when the water level is 10 m deep. (The volume of a cone is $V = \frac{1}{3}\pi r^2 h$)

9. (4 points) Find the absolute extrema of $f(x) = \dfrac{\ln x}{\sqrt{x}}$ on $\left[1, e^4\right]$.

10. (10 points) Given

$$f(x) = \frac{(2x+3)(x-3)^2}{x^3} = \frac{2x^3 - 9x^2 + 27}{x^3}, \quad f'(x) = \frac{9(x^2-9)}{x^4} \quad \text{and} \quad f''(x) = \frac{18(18-x^2)}{x^5}, \text{ find all:}$$

(a) x and y intercepts.

(b) Vertical and horizontal asymptotes.

(c) Intervals of which $f(x)$ is increasing or decreasing.

(d) Local (relative) extrema.

(e) Intervals of upward and downward concavity.

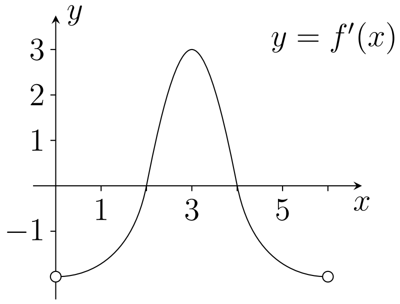
(f) Inflection points.

(g) Find the coordinates of the point(s) where the graph of f intersects its horizontal asymptote.

(h) Sketch the graph of $f(x)$. Label all intercepts, asymptotes, extrema, and points of inflection.

The fact that $f(3\sqrt{2}) \approx 0.23$ and $f(-3\sqrt{2}) \approx 3.77$ may also be useful.

11. (4 points) The graph below is of a function f' on $(0, 6)$.



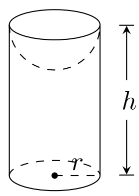
(a) Give the interval(s) where f is decreasing.

(b) Give the interval(s) where the graph of f is concave up.

(c) Give the x-coordinate(s) of the local (relative) maximum of f.

(d) Give the x-coordinate(s) of the point(s) of inflection of the graph of f.

12. (5 points) A closed cylindrical tank with a flat bottom and an inverted hemispherical top is to have a volume of $13\pi\,\text{m}^3$. Find the radius that will minimize the surface area of the tank. (The volume of a hemisphere of radius r is $\frac{2}{3}\pi r^3$ and its surface area is $2\pi r^2$.)



13. (3 points) Find $f(t)$ if $f''(t) = e^t - 3\cos(t) + 6t$, $f'(0) = 3$ and $f(0) = 1$.

14. (12 points) Evaluate each of the following integrals.

(a) $\displaystyle\int (6e^x - \sqrt[3]{x^7} + \pi^5)\,dx$

(b) $\displaystyle\int \frac{(x-1)^2}{x^3}\,dx$

(c) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin^2 x + \cos x}{\sin^2 x}\,dx$

(d) $\displaystyle\int_0^5 |x^2 - 9|\,dx$

15. (2 points) Find the derivative with respect to x of $y = \displaystyle\int_{\sqrt{x}}^{1} \frac{t}{t^2+1}\,dt$.

16. (a) (1 point) Express the integral $\displaystyle\int_0^3 (x^2 + 3)\,dx$ as a limit of Riemann sums.

(b) (3 points) Use summation formulæ and basic properties of limits to evaluate the integral from Part a.
Note that $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

No marks if you use the Fundamental Theorem of Calculus to evaluate the integral.

17. (1 point) Evaluate the integral $\displaystyle\int_{-2}^{0} \sqrt{4-x^2}\,dx$ by interpreting it in terms of area.

18. (1 point) If $\lim\limits_{x\to\infty} f'(x) = 0$, must the graph of f have a horizontal asymptote? Justify your answer.

Answers

1.(a)0 (b)∞ (c)DNE (d)0 (e)-1 (f)1 2.(a)$-\frac{20}{11}$ (b)-2 (c)$-\infty$ (d)0 (e)2

3.-2(removable), 5(jump), 12(infinite) 4.$\frac{-2x}{(x^2+1)^2}$ 5.(a)$5^{\cot x}\ln 5(-\csc^2 x) + 8x\sec(4x^2)\tan(4x^2)$

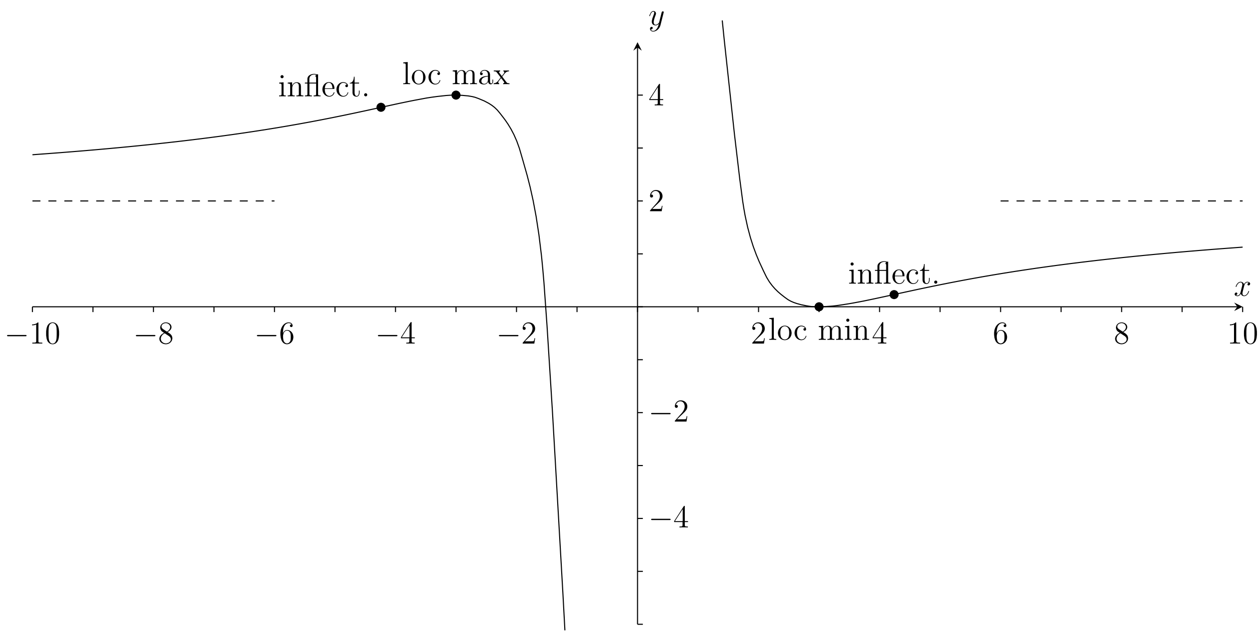
(b)$3e^x\tan^2(xe^x)\sec^2(xe^x)(x+1)$ (c)$\frac{1}{2}\sqrt{\frac{x^3\sin(2x)}{(x+1)^5}}\left(\frac{3}{x} + 2\cot(2x) - \frac{5}{x+1}\right)$ (d)$\frac{6x-ye^{xy}}{xe^{xy}-6y}$

(e)$\left(\frac{2x-3}{\cos x}\right)^x\left[x\left(\frac{2}{2x-3} + \tan x\right) + \ln\left(\frac{2x-3}{\cos x}\right)\right]$ 6.(b)$(1,3)$: $y=3$; $(1,-2)$: $y=2x-4$

7.(a)$f(x)$ is cont. on $[-1,4]$, $f(-1) = -4 < 0$ & $f(4) = 26 > 0$ (b)$\sqrt{\frac{13}{3}}$

(c)$f(x)$ is cont. & diff'able on $(-1,3)$, $f(-1) = -4 = f(3)$ 8.$\frac{4}{5\pi}$ m/min 9.Max:(1,0); Min:$(e^2, \frac{2}{e})$

10.



11.(a)(0,2), (4,6) (b)(0,3) (c)$x = 4$ (d)$x = 3$ 12. $3^{1/3}$ m 13.$f(t) = e^t + 3\cos t + t^3 + 2t - 3$

14.(a)$6e^x - \frac{3}{10}x^{10/3} + \pi^5 x + C$ (b)$\ln|x| + \frac{2}{x} - \frac{1}{2x^2} + C$ (c)$\frac{\pi}{4} - 1 + \sqrt{2}$ (d)$\frac{98}{3}$ 15.$-\frac{1}{2(x+1)}$

16.(a) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\frac{3}{n}$ (b)18 17. π

18. No. Consider $f(x) = \sqrt{x}$, which has no horizontal asymptote, but for which $\displaystyle\lim_{x\to\infty}\frac{1}{2\sqrt{x}} = 0$.