(Marks)

(3) 1. Use the graph of the function $f(x)$ to determine the following. Where appropriate, use ∞, $-\infty$, or "does not exist."

(a) $f(2) =$

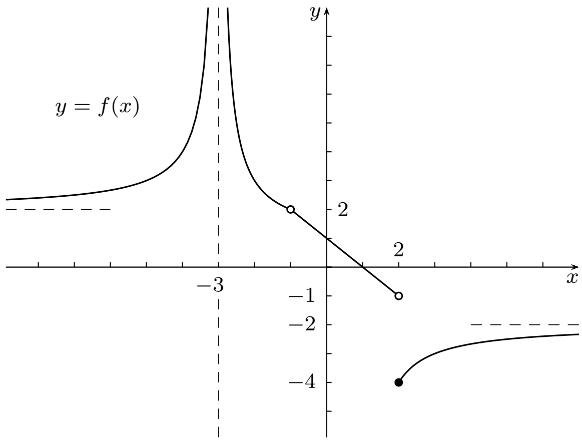
(b) $\lim_{x\to -3} f(x) =$

(c) $\lim_{x\to -1} f(x) =$

(d) $\lim_{x\to 2^-} f(x) =$

(e) $\lim_{x\to 2} f(x) =$

(f) $\lim_{x\to -\infty} f(x) =$



(10) 2. Evaluate the following. Where appropriate, use ∞, $-\infty$, or "does not exist."

(a) $\lim_{x\to 3} \dfrac{2x^2-5x-3}{x^2-8x+15}$

(b) $\lim_{\theta\to 0} \dfrac{\theta^2-\theta}{\tan(4\theta)}$

(c) $\lim_{x\to\infty} \sqrt{x^2+5x}-\sqrt{x^2+2x}$

(d) $\lim_{x\to -2} f(x)$ if the function satisfies $\dfrac{x^2-4}{x+2} \le f(x) \le x^2+5x+2$ for $x \neq -2$

(e) $\lim_{x\to 4^-} \dfrac{|x-4|}{(x-4)^2}$

(3) 3. For $\lim_{x\to 5} \dfrac{x^2-3x+k}{x^2-4x-5}$

(a) Find the value of k to make the limit exist and be finite.

(b) What is the value of the limit in that case?

(5) 4. Find all x-values at which $f(x)$ is discontinuous, and determine the type of each discontinuity at each value. Justify your answers.

$$f(x) = \begin{cases} \sqrt{1-x} & \text{if } x < -3 \\ \dfrac{x-3}{x} & \text{if } -3 < x \le 4 \\ \dfrac{x^2-16}{x^2-5x+4} & \text{if } x > 4 \end{cases}$$

(3) 5. Let $f(x) = \dfrac{4}{x-1}$

(a) Find all numbers c that satisfy the conclusion of the Mean Value Theorem for this function f on the interval $[2, 5]$.

(b) Show that there is no value of c that satisfies the conclusion of the Mean Value Theorem for this function f on the interval $[0, 2]$. Why does this not contradict the Mean Value Theorem?

(4) 6. Given the function $f(x) = \dfrac{x}{x+1}$, find $f'(x)$ using the **limit definition** of the derivative.

(Marks)

(15) 7. Find $\frac{dy}{dx}$ for each of the following:

(a) $y = 8x^7 + \sqrt[7]{x^8} - \log_8(x+7) + \frac{\sin(x^7)}{7} - 4^{3\pi} + e^{1/x}$

(b) $y = \frac{(2x+1)^5}{x^2-3}$

(c) $y = (x^3-1)^{\sec(x)}$

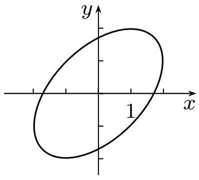
(d) $y = \tan^3(x)\csc(10x-1)$

(e) $y = \ln\left[\frac{(4x-1)(x^2+1)^{3/2}}{\sqrt{x}\,e^{4x}}\right]$

(3) 8. Find the 68^{th} derivative of $f(x) = 2^{2x} + \cos(x) - x^{67}$

(4) 9. For which values of x is the tangent line to $y = (x-5)^4(2x-1)^5$ horizontal?

(4) 10. The equation $x^2 - xy + y^2 = 3$ represents a "rotated ellipse," as shown below. Find the points at which the ellipse crosses the x-axis and show that the tangent lines at these points are parallel to each other.



(5) 11. A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the wall at a speed of 1.6 m/s, how fast is the height of his shadow on the wall changing when he is 4 m from the wall?

(10) 12. Given $f(x) = x\sqrt[3]{x+4}$ and $f'(x) = \frac{4(x+3)}{3(x+4)^{2/3}}$ and $f''(x) = \frac{4(x+6)}{9(x+4)^{5/3}}$, find all:

(a) x and y intercepts.

(b) Vertical and horizontal asymptotes.

(c) Intervals on which $f(x)$ is increasing or decreasing.

(d) Local (relative) extrema.

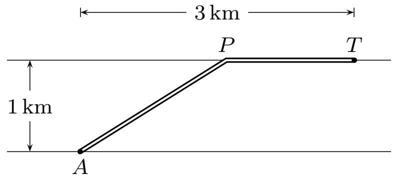
(e) Intervals of upward and downward concavity.

(f) Inflection point(s).

(g) On the next page sketch the graph of $f(x)$. Label all intercepts, asymptotes, extrema, and points of inflection.

(4) 13. Find the absolute extrema of $f(x) = (2x-1)\sqrt[3]{x}$ on $[-1, 1]$.

(5) 14. An oil company has a refinery at point A on the bank of a straight river 1 kilometer wide. It is going to run a pipe from point A to point P somewhere on the opposite side of the river, and then straight along the river to a tank T situated 3 kilometers downstream from A. It costs 15 thousand dollars per kilometer to run the pipe under the water and 9 thousand dollars per kilometer to run the pipe along the bank. What should be the distance from P to T in order to minimize the total cost of the pipe?



(Marks)

(9) 15. Evaluate the following integrals. (a) $\displaystyle\int \left(\frac{e^x}{4} + \frac{4^x}{e} + \frac{e}{4}\right)\, dx$ (b) $\displaystyle\int (2 + \sin x)\sec^2 x\, dx$

(c) $\displaystyle\int_{-\pi/2}^{\pi/2} (a \sin x + b \cos x)\, dx$

(3) 16. Find $f(x)$ given $f'(x) = \dfrac{2x^2 - 3x + 4}{x}$, $f(1) = 0$, and $x > 0$.

(4) 17. Sketch and shade the region bounded between the curve $y = \cos x$ and the x-axis from $x = 0$ to $x = \frac{3\pi}{2}$. Find the area of that region.

(2) 18. Express $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(\tfrac{2i}{n}\right) e^{\frac{2i}{n}} \left(\tfrac{2}{n}\right)$ as a definite integral (do not evaluate it).

(4) 19. Suppose $f(t)$ is a continuous function such that $\displaystyle\int_1^9 f(t)\, dt = 4$. Let $F(x) = \displaystyle\int_1^{x^2} f(t)\, dt$.

Find: (a) $F(1)$ (b)$F(3)$ (c) $F'(x)$

Answers

1.(a) -4 (b) ∞ (c) 2 (d) -1 (e) DNE (f) 2 2.(a) $-\frac{7}{2}$ (b) $-\frac{1}{4}$ (c) $\frac{3}{2}$

(d) -4 (e) ∞ 3.(a) $k = -10$ (b) $\frac{7}{6}$ 4. Infinite: $x = 0$; Removable: $x = -3$; Jump: $x = 4$

5.(a) $c = 3$ (b) $f'(c) \neq \frac{f(b)-f(a)}{b-a}$ because $\frac{-4}{(c-1)^2} \neq 4\ \forall c \in (0, 2)$ No contradiction: $f(1)$ discont.

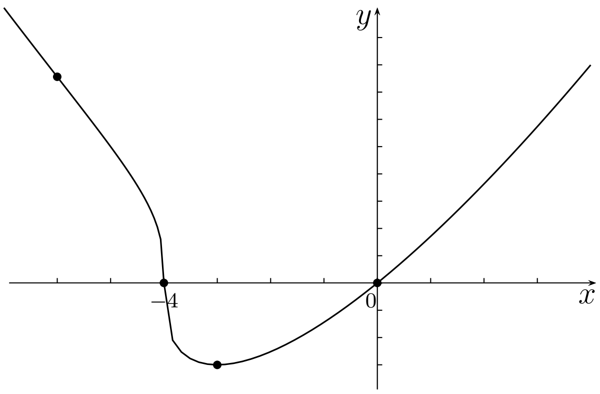
6. $f'(x) = \frac{1}{(x+1)^2}$ 7.(a) $56x^6 + \frac{8}{7}x^{1/7} - \frac{1}{(x+7)\ln 8} + x^6 \cos(x^7) - \frac{1}{x^2}e^{1/x}$ (b) $\frac{2(2x+1)^4(3x^2-x-15)}{(x^2-3)^2}$

(c) $(x^3 - 1)^{\sec x}[\sec x \tan x \ln(x^3 - 1) + \frac{3x^2}{x^3-1}\sec x]$

(d) $3\tan^2 x \sec^2 x \csc(10x - 1) - 10\tan^3 x \csc(10x - 1)\cot(10x - 1)$ (e) $\frac{4}{4x-1} + \frac{3x}{x^2+1} - \frac{1}{2x} - 4$

8. $2^{2x+68}(\ln 2)^{68} + \cos x$ 9. $x = 5$, $x = \frac{1}{2}$, $x = 3$ 10. $x = \pm\sqrt{3} \to y'(-\sqrt{3}) = 2$ and $y'(\sqrt{3}) = 2$

11. 0.6 m/s 12.



13. abs max $(-1, 3)$; abs min $(\frac{1}{8}, -\frac{3}{8})$ 14. $\frac{9}{4}$ km 15.(a) $\frac{e^x}{4} + \frac{4^x}{e\ln 4} + \frac{ex}{4} + C$ (b) $2\tan x + \sec x + C$

(c) $2b$ 16. $x^2 - 3x + 4\ln|x| + 2$ 17. 3 u^2 18. $\displaystyle\int_0^2 xe^x\, dx$ 19.(a) 0 (b) 4 (c) $f(x^2)2x$