1. Evaluate the following limits:
   (a) $\displaystyle\lim_{x\to\infty}\frac{5x^3-7x+12}{7x-4x^3}$ (b) $\displaystyle\lim_{x\to 2}\frac{\sqrt{x^2+5}-3}{x-2}$
   (c) $\displaystyle\lim_{x\to 0}\frac{\sin(4x)}{\tan(3x)}$ (d) $\displaystyle\lim_{x\to 1^+}\left(\frac{1}{x-1}+\frac{1}{x^2-3x+2}\right)$
   (e) $\displaystyle\lim_{x\to 1} f(x)$ if $2x-1\le f(x)\le x^2$ for $0<x<3$

2. The function $f(x)$ is defined by:

$$f(x)=\begin{cases} x^2+3 & \text{if } x\le -2 \\ -5x & \text{if } -2<x<5 \\ \dfrac{x^2}{x-6} & \text{if } x\ge 5 \end{cases}$$

   (a) Give all values of x where $f(x)$ is not continuous. State the type of discontinuity at each of these values and justify your answer.
   (b) At what values of x is $f(x)$ continuous but not differentiable ?

3. Given $f(x)=3-\dfrac{2}{x+1}$.
   (a) Find and simplify $\dfrac{f(2+h)-f(2)}{h}$.
   (b) Use the result of part (a) to find $f'(2)$.
   (c) Use the derivative rules to check your answer to part (b).
   (d) Find the equation of the normal line to the curve at $x=2$.

4. Find $\dfrac{dy}{dx}$ for each of the following:
   (a) $y=(2x-5)^4(x^2+3)^3$ (b) $y=\dfrac{3}{x^4}-\sqrt[4]{x^3}+4\log_3 x+3^{4x}-4e^3$
   (c) $x^2y^2+x\sin y=4\cos(3x)$ (d) $y=\ln\left(\dfrac{x^2\sqrt{\sin x}}{(2x-3)^3}\right)^4$
   (e) $y=(2x-1)^{\tan(3x)}$

5. For what value(s) of x will $f(x)=\dfrac{e^{2x}}{x-3}$ have a horizontal tangent line?

6. Find the critical numbers for $f(x)=\ln(x^4-8x^2+17)$

7. Find the absolute extrema for $f(x)=\dfrac{\sin x}{2+\cos x}$ on the interval $[0,\pi]$.

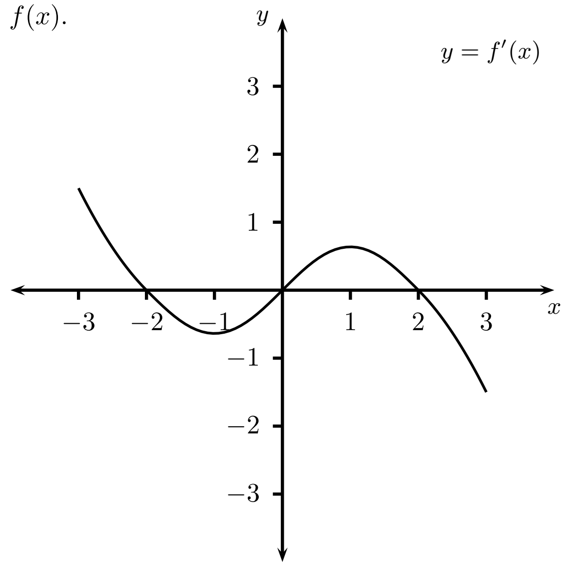
8. If $f(x)=\dfrac{e^{2x}}{e^{2x}-3}$, give the equations of all vertical and horizontal asymptotes of $f(x)$.

9. Given $f(x) = \frac{1}{3}\,x^{5/3}(8-x)$, $\quad f'(x) = \frac{8}{9}\,x^{2/3}(5-x)$ $\quad$ and $\quad f''(x) = \dfrac{40(2-x)}{27x^{1/3}}$

   Sketch the graph of $f(x)$ clearly showing all (if any) asymptotes, intercepts, local extrema and points of inflection.

10. In this question we are interested in an unknown function $f(x)$.

    The figure on the right shows the graph of $f'(x)$, the derivative of the unknown function, on the interval $[-3, 3]$.



    (a) Sketch the graph of $f''(x)$

    (b) Use these two graphs to find:

       (i) the interval(s) where $f(x)$ is increasing

       (ii) the interval(s) where $f(x)$ is concave down

       (iii) the critical values of $f(x)$

       (iv) the x-coordinates of the point(s) of inflection of $f(x)$

11. Sand is poured into a conical pile with the height of the pile always equalling its diameter. If the sand is poured at the constant rate of $5\,\mathrm{m}^3/\mathrm{s}$, at what rate is the height of the pile changing when the height is 2 meters? [$V = \frac{1}{3}\pi r^2 h$]

12. A cylindrical package to be sent by a postal service can have a maximum combined length and girth (perimeter of the circular cross section) of 84 inches. Find the dimensions of the package with the maximum volume.

13. The function $f(x)$ is defined by $\quad f(x) = x^3 + 3x - 2$.

    (a) Use the Intermediate Value Theorem to show that the equation $f(x) = 0$ has at least one real root.

    (b) Show that the equation $f(x) = 0$ has exactly one real root.

14. Evaluate the following:

    (a) $\displaystyle\int_1^2 \left(x + \frac{1}{x^2}\right)^2 dx$ $\qquad$ (b) $\displaystyle\int \left(3x^7 - \sqrt{x^7} + \frac{3^x}{7} - e^3\right) dx$

    (c) $\displaystyle\int \left(\frac{4\sin\theta}{\cos^2\theta}\right) d\theta$ $\qquad$ (d) $\displaystyle\int \frac{\left(\sqrt{t}+3\right)\left(\sqrt{t}-3\right)}{3t}\, dt$

15. Use both parts of the Fundamental Theorem of Calculus to find:

$$\frac{d}{dx}\left(\int_{\pi/2}^{x^2} \frac{\sin t}{t}\, dt\right) - \int_{\pi/2}^{x^2} \frac{d}{dt}\left(\frac{\sin t}{t}\right) dt$$

16. Given

$$f(x) = \begin{cases} |\,x+2\,| & \text{if } x \le 0 \\ \sqrt{4-x^2} & \text{if } 0 < x \le 2 \end{cases}$$

Evaluate $\displaystyle\int_{-5}^{2} f(x)\,dx$ by interpreting it in terms of areas.

17. Find the area under the curve $f(x) = 2 - \cos x$ and above the x-axis from $x = \frac{\pi}{6}$ to $x = \pi$.

18. Find an approximation for $\displaystyle\int_{2}^{10} (x^2 - 3x)\,dx$ using a Riemann sum with right endpoints and dividing the interval into 4 equal parts (i.e. using 4 rectangles.)

ANSWERS

1. (a) $-5/4$ (b) $2/3$ (c) $4/3$ (d) -1 (e) 1
2. (a)
   - $x = 2$, jump discontinuity; one-sided limits at 2 do not equal each other.
   - $x = 6$, infinite discontinuity; right-sided limit at 6 equals ∞.

   (b) $x = 5$
3. (a) $\dfrac{2}{3(3+h)}$ (b) $f'(2) = \lim\limits_{h\to 0} \dfrac{2}{3(3+h)} = \dfrac{2}{9}$

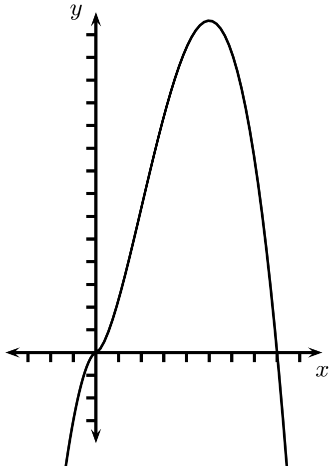
   (c) $f'(x) = \dfrac{2}{(x+1)^2}$, thus $f'(2) = \dfrac{2}{9}$ (d) $y = -\dfrac{9}{2}x + \dfrac{34}{3}$
4. (a) $2(2x-5)^3(x^2+3)^2(10x^2-15x+12)$

   (b) $-\dfrac{12}{x^5} - \dfrac{3}{4x^{1/4}} + \dfrac{4}{(\ln 3)x} + 4(\ln 3)3^{4x}$

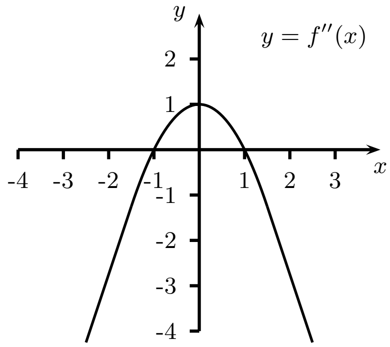
   (c) $\dfrac{-12\sin(3x) - \sin(y) - 2xy^2}{2x^2y + x\cos y}$

   (d) $\dfrac{8}{x} + \dfrac{2\cos x}{\sin x} - \dfrac{24}{2x-3}$

   (e) $y\left[3\sec^2(3x)\ln(2x-1) + \dfrac{2\tan(3x)}{2x-1}\right]$
5. $7/2$
6. $-2, 0, 2$
7. Absolute maximum is $1/\sqrt{3}$, absolute minimum is 0.
8. Horizontal asymptotes: $y = 0$, $y = 1$. Vertical asymptote: $x = \ln\sqrt{3}$.

9. 

- x-intercepts : $(0,0),\ (8,0)$
- y-intercept: $(0,0)$
- Absolute maximum of $5\sqrt[3]{25}$ occurs at $x=5$.
- Inflection points: $(0,0),\ (2,4\sqrt[3]{4})$

10. (a) 

(b) (i) $(-3,-2),\ (0,2)$
(ii) $(-3,-1),\ (1,3)$
(iii) $-2,0,2$
(iv) $-1,1$

11. $5/\pi$ m/s
12. Length: 28 inches, girth: 56 inches.
13. (a) Apply the Intermediate Value Theorem with $f(x)$ and the interval $[0,1]$. (Details omitted).
    (b) Note that $f'(x) = 3x^2 + 3 > 0$ for all x, thus $f(x)$ is always increasing. This implies $f(x) = 0$ has only the one solution.
14. (a) $\dfrac{21}{8} + 2\ln 2$ (b) $\dfrac{3}{8}x^8 - \dfrac{2}{9}\sqrt{x^9} + \dfrac{3^x}{7\ln 3} - e^3 x + C$
    (c) $4\sec\theta + C$ (d) $\dfrac{t}{3} - 3\ln|t| + C$
15. $\dfrac{\sin(x^2)}{x^2}(2x-1) + \dfrac{2}{\pi}$
16. $\pi + \dfrac{13}{2}$
17. $\dfrac{5}{3}\pi + \dfrac{1}{2}$
18. 360