(Marks)

1. Evaluate the limit or explain why it does not exist. Use $+\infty$, $-\infty$ or "does not exist" where appropriate.

(2) (a) $\displaystyle\lim_{x\to -3} \frac{x^2+x-6}{x^3+3x^2-5x-15}$

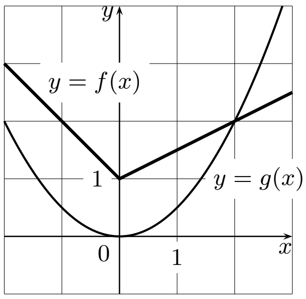
(2) (b) $\displaystyle\lim_{\theta\to 0} \frac{\sin\theta}{\sqrt{\theta+2}-\sqrt{2}}$

(2) (c) $\displaystyle\lim_{x\to \infty} \frac{3+2x+5x^2-2x^3}{3x^2+x+7}$

(2) (d) $\displaystyle\lim_{x\to -\infty} \frac{6x+1}{\sqrt{4x^2-3}}$

(4) 2. Find a value of c such that $g(x) = \begin{cases} \sqrt{cx-1}, & \text{if } x \le 5 \\ \dfrac{200}{x^2}, & \text{if } x > 5 \end{cases}$ will be continuous at $x = 5$.

(3) 3. Given the following graphs, evaluate the following if possible. Use $+\infty$, $-\infty$ or "does not exist" where appropriate. Assume that $g'(1) = 1$ and that $g(1) = \frac{1}{2}$.



(a) $\displaystyle\lim_{x\to 0} f(x)$

(b) $\displaystyle\lim_{x\to 0^-} \frac{f(x)}{g(x)}$

(c) $g'(0)$

(d) $f'(0)$

(e) $\displaystyle\left(\frac{f}{g}\right)'(1)$

(f) $(f\circ g)'(1)$

(3) 4. Does the function $R(x) = \dfrac{x^3}{1000} - \dfrac{100}{x^2} + \dfrac{x}{10}$ have a zero in the open interval $(1, 10)$? Explain your answer.

(4) 5. Given the function $f(x) = \dfrac{x}{x+1}$, find $f'(x)$ using the LIMIT DEFINITION of the derivative.

6. Find $\dfrac{dy}{dx}$ for each of the following:

(3) (a) $y = x^2 + 2^x + \ln|x| - \dfrac{1}{2x} + \sqrt[3]{x^2} + \sqrt{e^{2\pi}}$

(3) (b) $y = \sin(2x-3)^6 - \cos^6(2x-3)$

(3) (c) $y = \left(\dfrac{3x+4}{5x^2+1}\right)^3$

(3) (d) $e^{xy} = 17x - \tan(y)$

(3) (e) $y = (2x+3)^{2x+3}$

(3) (f) $y = \ln\left(\dfrac{(5x+2)^2 e^{5x}}{(2-\sqrt{x})^{2/3}}\right)$

(Marks)

(4) 7. Determine for which values of x the function $g(x) = (x-3)^5(3x+4)^3$ has a horizontal tangent line.

(4) 8. Find an equation for the tangent line to $y = \dfrac{3x+5}{x^2+3}$ when $x = 1$.

9. Given the curve $x^2 + xy + y^2 = 4$:

(2) (a) Find $\dfrac{dy}{dx}$.

(2) (b) Determine all points (x, y) on the curve where the tangent line is parallel to the line $y = x + 4$.

(1) 10. (a) State the product rule for derivatives.

(2) (b) Use logarithmic differentiation to prove the product rule for a function $y = f(x)g(x)$.

(5) 11. A new hydro-electric dam is built on Algonquin land in Parc de La Vérendrye. When the dam is finally closed, the flood waters spread outward in the form of a semi-circle centered at the middle of the dam, at a rate of 800,000 m^2 per hour. At what rate is the radius of the flooded land increasing when 2,000,000 m^2 of traditional native hunting grounds have been covered?

12. Given $f(x) = \dfrac{2+x-x^2}{(x-1)^2}$, $f'(x) = \dfrac{x-5}{(x-1)^3}$ and $f''(x) = \dfrac{14-2x}{(x-1)^4}$:

(2) (a) Find all vertical and horizontal asymptotes.

(1) (b) Find the intervals of increase and decrease.

(1) (c) Find all local (relative) extrema.

(1) (d) Find the intervals of upward and downward concavity.

(1) (e) Find all inflection points.

(3) (f) Sketch the graph of f. Label all intercepts, asymptotes, extrema and points of inflection.

(1) (g) Identify the absolute maximum and absolute minimum values of $f(x)$ if they exist.

(5) 13. A rectangular cage (called a battery cage) for a laying hen has a volume of 0.016 m^3. While the European Union will have phased out battery cages by 2012 and Germany has already banned them, in Canada 98% of all hens are housed in battery cages. If the material for the base of the cage costs \$2 per m^2 and the material for the sides and the top costs \$3 per m^2, then what would be the dimensions of a lowest-cost battery cage with height 0.4 m?

(4) 14. Find the absolute maximum and absolute minimum of the function $f(x) = 5x^{2/3} - x^{5/3}$ on the closed interval $[-1, 4]$.

15. Consider the definite integral $\displaystyle\int_1^3 (4x^2+1)\,dx$.

(2) (a) Find an approximation to the value of the integral using a Riemann sum with right endpoints and 4 rectangles.

(1) (b) Express the definite integral $\displaystyle\int_1^3 (4x^2+1)\,dx$ as a limit of Riemann sums. Do not evaluate the limit.

(4) 16. Find the position function $s(t)$ of a moving particle which has an acceleration function $a(t) = 12t^2 - 3\sin t$, an initial velocity of $v(0) = 0$ m/sec. and an initial position of $s(0) = 3$ m.

17. Evaluate the following integrals.

(2) (a) $\displaystyle\int (x^5 + \sqrt[5]{x^2} - 5^x + 5^2)\,dx$

(Marks)

(3) (b) $\displaystyle\int_1^4 \frac{(x+2)^2}{\sqrt{x}}\,dx$

(1) (c) $\displaystyle\int \frac{d}{dx}\sqrt{x^3+5}\,dx$

(2) (d) $\displaystyle\int \frac{3\sin^2 x - 2}{\sin^2 x}\,dx$

(3) 18. Find the area between $y = 2 + \dfrac{3}{x}$, $x = 1$, $x = e$ and the x-axis.

19. Let $F(x) = \displaystyle\int_2^{\sqrt{x}} \sqrt{t^3+1}\,dt$.

(1) (a) Evaluate $F(4)$.

(2) (b) Evaluate $F'(x)$ using the Fundamental Theorem of Calculus.

SOLUTIONS 1. a) $-5/4$ b) $2\sqrt{2}$ c) $-\infty$ d) -3 2. $c = 13$ 3. a) 1 b) $+\infty$ c) 0 d) dne e) -5 f) 1/2 4. Yes, by the Intermediate Value Theorem 5. $f'(x) = \dfrac{1}{(1+x)^2}$ 6. a) $\frac{dy}{dx} = 2x + 2^x \ln 2 + \frac{1}{x} + \frac{1}{2x^2} + \frac{2}{3x^{1/3}}$ b) $\frac{dy}{dx} = \cos(2x-3)^6 \cdot 6(2x-3)^5 \cdot 2 - 6\cos^5(2x-3)(-\sin(2x-3)) \cdot 2$ c) $\frac{dy}{dx} = 3(\frac{3x+4}{5x^2+1})^2 \cdot \frac{(5x^2+1)3-(3x+4)10x}{(5x^2+1)^2}$ d) $\frac{dy}{dx} = \frac{17-ye^{xy}}{xe^{xy}+\sec^2 y}$ e) $\frac{dy}{dx} = (2x+3)^{2x+3}(2+2\ln(2x+3))$ f) $\frac{dy}{dx} = \frac{10}{5x+2} + 5 + \frac{1}{3\sqrt{x}(2-\sqrt{x})}$ 7. $x =$-4/3, 3, 7/24 8. $y - 2 = \frac{-1}{4}(x-1)$ 9.a) $\frac{dy}{dx} = \frac{-2x-y}{x+2y}$ b) $(2,-2)$ and $(-2,2)$ 11. $\frac{400}{\sqrt{\pi}}$ m/hour 12. a) VA: x=1 and HA: y= -1 b) increasing: $(\infty,1) \cup (5,\infty)$, decreasing: $(1,5)$ c) local minimum: $(5,\frac{-9}{8})$ d) CU: $(-\infty,1) \cup (1,7)$ CD: $(7,\infty)$ e) IP at $(7,\frac{-10}{9}$ f) see picture below g) no global max, global minimum: $(5,\frac{-9}{8})$

13. 20 cm × 20 cm × 40 cm 14. absolute max: (-1,6) absolute min: (0,0) 15. a) 45 b) $\displaystyle\lim_{x\to\infty} \frac{2}{n}\sum_{i=1}^{n}[4(1+\tfrac{2i}{n})^2+1]$ 16. $s(t) = t^4 + 3\sin t - 3t + 3$ 17. a) $\frac{x^6}{6} + \frac{5x^{7/5}}{7} - \frac{5^x}{\ln 5} + 25x + C$ b) $\frac{586}{15}$ c) $\sqrt{x^3+5} + C$ d) $3x + 2\cot x + C$ 18. $2e+1$ 19. a) 0 b) $\sqrt{(\sqrt{x})^3+1} \cdot \frac{1}{2\sqrt{x}}$