1. Let $y = \frac{x}{2}\sqrt{9-4x^2} + \frac{9}{4}\arcsin\left(\frac{2x}{3}\right)$. Find $\frac{dy}{dx}$ and **simplify** your answer.

2. Evaluate the integrals. **Exact answers only**: no decimals!

a) $\int (2t+1)\sqrt{t+1}\,dt$ **b)** $\int e^{3x}\sin(4x)\,dx$ **c)** $\int \frac{\sin^3\left(\frac{1}{x}\right)\cos^2\left(\frac{1}{x}\right)}{x^2}\,dx$

d) $\int \frac{11x^2-14x+8}{(x^2+1)(2x-1)}\,dx$ **e)** $\int \frac{1}{x^3\sqrt{4x^2-9}}\,dx$ **f)** $\int_0^{\frac{\pi}{4}} \frac{\sec(x)\tan(x)}{\sqrt{4-\sec^2(x)}}\,dx$

3. Determine the value of the limits. In each case **justify** your answer by showing all relevant steps and using proper mathematical notation.

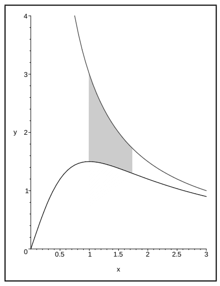
a) $\lim_{x\to 0} \frac{\sin^2(2x)}{\cos^3(3x)-1}$ **b)** $\lim_{x\to 1^+} x^{\left(\frac{2}{x-1}\right)}$

4. Determine whether the improper integrals converge or diverge; if an integral converges, give its **exact** value. Use correct mathematical notation throughout.

a) $\int_{-\infty}^{2} \frac{3}{x^2+4}\,dx$ **b)** $\int_1^3 \frac{3}{x^2-x-2}\,dx$

5. Compute the area of the region bounded by $y = x^2$ and $y = \sqrt{8x}$. **Simplified exact answer only**: no decimals!

6. Let $\mathcal{R}$ be the region between the graphs of $y = \frac{3}{x}$ and $y = \frac{3x}{x^2+1}$ from $x = 1$ to $x = \sqrt{3}$ as shown in the figure.



a) Set up, but **do not evaluate**, the integral required to find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis

b) Find the volume of the solid generated by revolving $\mathcal{R}$ about the y-axis. **Simplified exact** answer only.

7. Solve for the differential equation: $\frac{dy}{dx} = y\sin(x)$ subject to $y\left(\frac{\pi}{2}\right) = 2$. An **explicit** solution $y(x) = \cdots$, please!

8. Determine whether the sequence converges or diverges. **Justify**: if it converges give its limit, or explain why it diverges.

a) $a_n = \left(1 - \frac{1}{3n}\right)^{5n}$ **b)** $b_n = \left(\frac{1}{3n} - 1\right)^{5n}$

9. Find the sum of the series. **Exact answers only**: no decimals!

a) $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$ **b)** $\sum_{n=1}^{\infty} \frac{3^n+4^{n+1}}{5^n}$ **c)** $\sum_{n=0}^{\infty} \frac{\sin^n(x)}{3^n}$

10. Determine whether the series converge or diverge. **Justify**: state the test that you use, and verify that the conditions for using the test are satisfied.

a) $\sum_{n=1}^{\infty} \frac{n^n}{3^n n!}$ **b)** $\sum_{n=1}^{\infty} \frac{1}{3^n+\cos^2(n)}$ **c)** $\sum_{k=1}^{\infty} \left(\frac{3k^3+1}{2k^3+1}\right)^k$ **d)** $\sum_{n=2}^{\infty} \sin\left(\frac{1}{n^2}\right)$

11. Determine whether the series is absolutely convergent, conditionally convergent or divergent. **Justify**: state the test that you use.

a) $\sum_{k=2}^{\infty} (-1)^k \frac{1}{k\ln(k)}$ **b)** $\sum_{k=0}^{\infty} (-2)^k \frac{k+1}{5^k}$

12. Determine the radius and the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2n-1}{2^n(n+1)}(x-2)^n$.

13. **a)** Find the first 4 terms of the Taylor series for $f(x) = \sqrt{x+1}$ centered at $a = 3$.

b) Find the n^{th} term of the above Taylor series.

Answers

1. $\dfrac{dy}{dx} = \sqrt{9 - 4x^2}$

2. a) (substitution) $\rightarrow\ 2\left(\dfrac{2}{5}(t+1)^{5/2} - \dfrac{1}{3}(t+1)^{3/2}\right) + C$

b) (by parts) $\rightarrow\ -\dfrac{4}{25}e^{3x}\cos(4x) + \dfrac{3}{25}e^{3x}\sin(4x) + C$

c) (substitution and trigo) $\rightarrow\ \dfrac{1}{3}\cos^3\left(\dfrac{1}{x}\right) - \dfrac{1}{5}\cos^5\left(\dfrac{1}{x}\right) + C$

d) (partial fractions) $\rightarrow\ \displaystyle\int \frac{4x-5}{x^2+1} + \frac{3}{2x-1}\,dx = 2\ln(x^2+1) - 5\arctan(x) + \frac{3}{2}\ln\mid 2x-1 \mid + C$

e) (inverse substitution) $\rightarrow\ \dfrac{1}{18}\dfrac{\sqrt{4x^2-9}}{x^2} + \dfrac{2}{27}\operatorname{arcsec}\left(\dfrac{2x}{3}\right) + C$

f) (substitution) $\rightarrow\ \dfrac{\pi}{12}$

3. a) (l'Hopital and limit rules) $\rightarrow\ -\dfrac{8}{27}$

b) (l'Hopital and log) $\rightarrow\ e^2$

4. a) (limit of inverse trigo) $\rightarrow\ \dfrac{9\pi}{8}$

b) (limit of log) $\displaystyle\lim_{t\to 2^-}\int_1^t \frac{1}{x-2}\,dx - \lim_{s\to 2^+}\int_s^3 \frac{1}{x-1}\,dx \ \rightarrow\ div$

5. $\displaystyle\int_0^2 (\sqrt{8x} - x^2)\,dx = \frac{8}{3}$

6. a) $\displaystyle V = \pi\int_1^{\sqrt{3}} \left\{\left(\frac{3}{x}\right)^2 - \left(\frac{3x}{x^2+1}\right)^2\right\} dx$

b) $\displaystyle V = 2\pi\int_1^{\sqrt{3}} x\left(\frac{3}{x} - \frac{3x}{x^2+1}\right) dx = 6\pi\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\pi^2}{2}$

7. $y(x) = 2e^{-\cos(x)}$

8. a) $\displaystyle\lim_{n\to\infty} a_n = e^{-5/3}$

b) $b_n = (-1)^n a_n$ diverges

9. a) $\displaystyle\frac{1}{2}\sum_{n=1}^{\infty}\left\{\frac{1}{n} - \frac{1}{n+2}\right\} = \frac{3}{4}$

b) $\dfrac{35}{2}$

c) $\dfrac{3}{3 - \sin(x)}$

10. a) converges (ratio test)

b) converges (comparison with $\displaystyle\sum_{n=1}^{\infty}\frac{1}{3^n}$)

c) divergent (root test or divergence test)

d) converges (limit comparison with $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n^2}$)

11. a) conditionally convergent (alternating series test, integral test)$= (\approx 0.5264, div)$

b) absolutely convergent (ratio test) $\left(\dfrac{25}{49}, \dfrac{25}{9}\right)$

12. (ratio test) $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}|x-2| \ \rightarrow\ R = 2$ and $0 < x < 4$

13. a) $\sqrt{x+1} = 2 + \dfrac{1}{4}(x-3) - \dfrac{1}{64}(x-3)^2 + \dfrac{1}{512}(x-3)^3 + \cdots$

b) $(-1)^{n+1}\dfrac{1\cdot 3\cdot 5\cdots(2n-3)}{2^{3n-1}n!}(x-3)^n$