(Marks)

1. Evaluate the following limits showing all important steps and using mathematical notation correctly.

   (a) $\displaystyle\lim_{x\to 2}\frac{2x^2-7x+6}{2x^2+x-10}$ (b) $\displaystyle\lim_{x\to 4}\frac{\sqrt{x+12}-4}{x-4}$ (c) $\displaystyle\lim_{x\to -\infty}\frac{x+3}{\sqrt{2x^2+5x}}$ (d) $\displaystyle\lim_{x\to 5^-}\frac{|2x-10|}{3x-15}$

2. Using the *limit definition of derivative*, show that the derivative of $f(x)=\dfrac{1}{x+2}$ is $\dfrac{-1}{(x+2)^2}$.

3. Consider the function $f(x)=\dfrac{x}{2x^2+x}$.

   (a) State the two values of x where f is not continuous, and in each case give a reason why.

   (b) Evaluate the limit of f at each of these values, and state the type of discontinuity at each point.

4. Sketch the graph of a function having all of the following properties.

   - $f(x)$ is continuous everywhere.
   - $f(x)$ is differentiable everywhere except at $x=\pm 2$.
   - $f'(x)<0$ only when $|x|<2$.

5. Find the derivative for each of the following functions. *It is not necessary to simplify your answers.*

   (a) $f(x)=3x^4-\dfrac{4}{x^3}+\sqrt[3]{x^4}-\log_3(4x)+4^3$ (b) $f(x)=\left(\dfrac{x}{x^3-2}\right)^5$

   (c) $f(x)=\mathrm{e}^{\tan x}\sec 3x$ (d) $f(x)=\sqrt{1+\sqrt{1+x^2}}$

6. Find y' for each of the following.

   (a) $y=(\sin x)^{x^2+1}$ (b) $y=\dfrac{(x-4)^3}{x^5\sqrt[3]{x^3+1}}$

7. Find all points where the graph of $f(x)=\mathrm{e}^x(x+1)^2$ has horizontal tangents.

8. Find the $\mathbf{15^{th}}$ derivative of $f(x)=\sin(x)+e^{-x}+x^{14}$.

9. Given the curve $x^2-xy+y^2=9$.

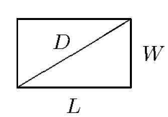
   (a) Find an expression for the derivative, $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

   (b) Find an equation for the tangent line which touches the curve at $(3,0)$.

10. A rectangle of length L and width W has a constant area of 800 m^2 even though L is increasing by 2 m/s.

    (a) Let D be the length of a diagonal. Find the rate of change of D when $W=10$ m.

    (b) Find the width W, at the instant when it is decreasing at a rate of 1 m/s.



11. Given $f(x)=\dfrac{2+x-x^2}{(x-1)^2}$ $f'(x)=\dfrac{x-5}{(x-1)^3}$ and $f''(x)=\dfrac{2(7-x)}{(x-1)^4}$: find (if any)

    (a) all x and y intercepts,

    (b) equations of horizontal and vertical asymptotes,

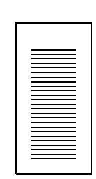
    (c) intervals where $f(x)$ is increasing, decreasing, concave up and concave down,

    (d) relative extrema and points of inflection.

    Make a neat sketch of the graph that clearly illustrates all the above-mentioned features.

12. Find the absolute maximum and the absolute minimum of $f(x)=\cos x+x\sin x$ on the interval $[0,\pi]$.

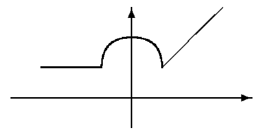
13. A rectangular page is to contain 45 square inches of printed material. The margins on all sides except the top are 1 in wide. At the top the margin is 1.5 in. Find the dimensions of the page such that the least amount of paper is used.



(Marks)

14. Find f if $f''(x) = -\cos x + e^x + \dfrac{15}{4}\sqrt{x}$, $\quad f'(0) = 1$ and $f(0) = 7$.

15. Let $f(x) = \begin{cases} 1 & x < -1 \\ 1 + \sqrt{1 - x^2} & -1 \le x \le 1 \\ x & x > 1 \end{cases}$



Evaluate the integral $\displaystyle\int_{-2}^{2} f(x)\,dx$ by interpreting it in terms of areas.

16. Let $\mathcal{S}$ be the region bounded by $f(x) = \dfrac{3 + x}{x}$ and the x-axis between $x = 1$ and $x = 9$.

   (a) Approximate the value for the area of $\mathcal{S}$ by finding the Riemann sum with 4 equal subintervals, taking the sample points to be the midpoints.

   (b) Find the exact value of the area. (*Use whatever method you like.*)

17. Use the Fundamental Theorem of Calculus to find $g'(x)$ where $g(x) = \displaystyle\int_1^{\sin x} \frac{1 - t^2}{t}\,dt$.

18. Evaluate

   (a) $\displaystyle\int (e^x \cos x - \tan x)\sec x\,dx$ (b) $\displaystyle\int \frac{6 + x^{2/3} - 4x^5}{x}\,dx$ (c) $\displaystyle\int_{-1}^{3} (x^2 + 2x)\,dx$

ANSWERS

1. (a) 1/9, (b) 1/8, (c) $-1/\sqrt{2}$, (d) -2/3 3. $\lim_{x\to 0} f(x) = 1$, removable; $\lim_{x\to -1/2} f(x)$ dne, infinite. 4. See below.

5. (a) $12x^3 + 12x^{-4} + \frac{4}{3}x^{1/3} + \dfrac{4}{4x\ln 3}$ (b) $5\left(\dfrac{x}{x^3-2}\right)^4 \left[\dfrac{(x^3-2) - 3x^3}{(x^3-2)^2}\right]$ (c) $e^{\tan x}\sec^2 x \sec 3x + 3e^{\tan x}\sec 3x \tan 3x$

(d) $\dfrac{1}{2\sqrt{1+\sqrt{1+x^2}}}\dfrac{2x}{2\sqrt{1+x^2}}$ 6. (a) $y' = y\left(2x\ln(\sin x) + (x^2+1)\dfrac{\cos x}{\sin x}\right)$ (b) $y' = y\left(\dfrac{3}{x-4} - \dfrac{5}{x} - \dfrac{3x^2}{3(x^3+1)}\right)$

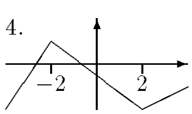
7. $(-1, 0)$; $(-3, 4/e^3)$ 8. $f^{(15)}(x) = -\cos x - e^{-x}$ 9. (b) $y = 2x - 6$ 10.(a) $\dfrac{63}{4\sqrt{65}}$ m/s (b) 20 m

11. Intercepts $(-1, 0)$; $(2, 0)$; $(0, 2)$ minimum $(5, -9/8)$ inflection $(7, -10/9)$ asymptotes $x = 1$; $y = -1$. See graph below.

12. Max $(\pi/2, \pi/2)$; min $(\pi, -1)$ 13. 8×10 in 14. $f(x) = \cos x + e^x + x^{5/2} + 5$ 15. $\dfrac{9+\pi}{2}$

16. (a) 57/4, (b) $8 + 3\ln 9$ 17. $\left(\dfrac{1 - \sin^2 x}{\sin x}\right)\cos x$ 18. (a) $e^x - \sec x + C$ (b) $6\ln|x| + \frac{3}{2}x^{2/3} - \frac{4}{5}x^5 + C$ (c) 52/3

4.



11.