INSTRUCTIONS:

1. Write your name on this sheet and on the first page of each booklet you use. Number the booklets if you use more than one.
2. Answer all questions in the booklet(s) (use both sides of each page) and show all supporting work. Use correct notation at all times.
3. When you submit your answers, place this question sheet (and any additional booklets you use) inside the first page of your first booklet.
4. Remember that *the use of graphing/programmable calculators is not permitted.*

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(4) 1. Differentiate

$$y = \sqrt{x}\arcsin\sqrt{x} + \sqrt{1-x}$$

with respect to x and simplify your answer.

(35) 2. Evaluate the integrals.

(a) $\displaystyle\int_0^1 \frac{e^{\arctan x}}{1+x^2}\,dx$

(b) $\displaystyle\int \frac{x\,dx}{\sqrt{2x-1}}$

(c) $\displaystyle\int e^{5x}\cos x\,dx$

(d) $\displaystyle\int \sin^5 3x\,dx$

(e) $\displaystyle\int \tan^3\left(\tfrac{1}{2}x\right)\sec^3\left(\tfrac{1}{2}x\right)\,dx$

(f) $\displaystyle\int \frac{x^2\,dx}{\sqrt{9-x^2}}$

(g) $\displaystyle\int \frac{x^2-4x+5}{(x+1)(x^2+4)}\,dx$

(8) 3. Evaluate the improper integrals.

(a) $\displaystyle\int_0^\infty xe^{-x^2}\,dx$

(b) $\displaystyle\int_1^2 \frac{dx}{x\sqrt{x^2-1}}$

(9) 4. Evaluate the limits.

(a) $\displaystyle\lim_{x\to\infty}(\ln x)^{e^{-x}}$

(b) $\displaystyle\lim_{x\to 0}\frac{x-x\cos x}{\sin x - x}$

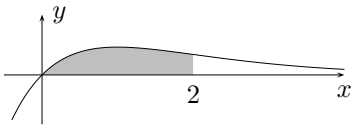
(c) $\displaystyle\lim_{x\to\infty}\left\{\ln(x+1)-\ln(2x+3)\right\}$

(4) 5. Compute the area of the region bounded by the graphs of

$$y = 4x - x^2 \quad \text{and} \quad y = 8 - 2x.$$

6. Let $\mathscr{R}$ be the shaded region in the figure, which is bounded by the graphs of

$$y = xe^{-x}, \quad y = 0, \quad x = 0, \quad \text{and} \quad x = 2.$$



(6) (a) Set up the integrals required to compute the volume of the solid obtained by rotating $\mathscr{R}$ about: (i) the x-axis, (ii) the y-axis.

(3) (b) Evaluate *one* of the integrals from part (a).

(4) 7. Solve the differential equation

$$yy' = (x+3)(y^2+3); \quad y(-6) = -2.$$

(4) 8. Determine whether the series converges or diverges; if it converges, find the sum. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \big\{\arctan n - \arctan(n+1)\big\}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2^{2n}}{3^{n+1}}$

(6) 9. Determine whether the series converges or diverges. State the tests you use and verify that the conditions for using them are safisfied.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+2}{2n+1}\right)^n$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{e^k}{k}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{\operatorname{arcsec} k}{k}$

(9) 10. Label the series as absolutely convergent, conditionally convergent, or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{5^n}{(n+1)!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n+1}}{n^2}$

(4) 11. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{n2^n}.$$

(4) 12. Find the Taylor series of $f(x) = \ln x$ centred at 1.