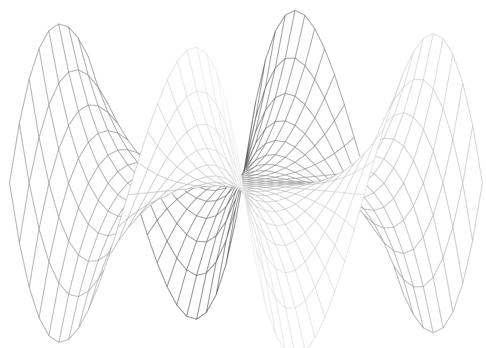
CEGEP JOHN ABBOTT COLLEGE

DEPARTMENT OF MATHEMATICS

FINAL EXAM

MAY 2005

CALCULUS III
201–DDB



INSTRUCTORS: F. Lo Vasco, R.A.G. Seely

INSTRUCTIONS:

1. Write all of your solutions in the booklet provided and show your supporting work. Be sure to put your name on the booklet in the space provided.
2. Write on both sides of the pages in the booklet; if you need more paper, ask for another booklet, and be sure to put your name on all booklets handed in.
3. Check that this exam contains 2 numbered pages, excluding the cover page.

(Marks)

(7) 1. Let $f'(x) = \sum_{n=2}^{\infty} \frac{3^n(x-5)^n}{n!}$. (*This is not a typo! It really does say* f'.)

(a) Find a power series for $f(x)$ given that $f(5) = 1$.

(b) Find the radius and interval of convergence for $f(x)$.

(c) Find $f^{(7)}(5)$.

(8) 2. Find the Maclaurin series for each of the following functions; in each case give the radius of convergence of the series.

(a) $\int_0^x \frac{\arctan t}{t}\, dt$

(b) $\frac{x^2}{x-2}$

(6) 3. Find the Taylor polynomial of degree 2 for $f(x) = \sqrt[4]{x}$ centered at $x = 16$; use it to estimate the value of $\sqrt[4]{15}$, and give a bound on the error of your estimate. Justify your calculations.

(14) 4. Consider the curve given by parametric equations $\begin{cases} x = 3t - t^3 \\ y = 3t^2 \end{cases}$

(a) Find the x and y intercepts and the coordinates of the point where the tangent is horizontal and where it is vertical.

(b) Find $\frac{d^2y}{dx^2}$ and deduce where the graph changes concavity.

(c) Sketch the graph of the curve, showing these details.

(d) The graph forms a loop; find the length of the loop.

(e) Find the area of the loop.

(8) 5. (a) Sketch (on the same axes) the graphs of $r = 2 - 2\sin\theta$ and $r = 2\sin\theta$. Find all points of intersection.

(b) Set up (*but do not evaluate*) the integrals needed to find
(i) the area of the region **inside both** $r = 2 - 2\sin\theta$ and $r = 2\sin\theta$.
(ii) the length of the part of the curve $r = 2 - 2\sin\theta$ which lies **outside** $r = 2\sin\theta$.

(8) 6. Identify and sketch the following surfaces. Show all your work, including appropriate traces and intercepts.

(a) $z^2 = 9 - 4x^2 - y^2$

(b) $z^2 = r^2 - 9$

(3) 7. What is the equation in spherical coordinates of the curve given by $(x^2+y^2+z^2)^2 = 2z(x^2+y^2)$?

(Marks)

(10) 8. A curve $\mathcal{C}$ has the vector equation $\boldsymbol{r}(t) = \langle t, t^2, t \rangle$

(a) Sketch (and describe) the curve.

(b) Find the parametric equations of the tangent line to the curve at the point where $\mathcal{C}$ intersects the plane $x + y + z + 1 = 0$.

(c) Find the angle that the tangent line makes with the plane.

(d) Find the tangential and normal components of acceleration a_T, a_N of a particle moving along $\mathcal{C}$.

(4) 9. Find the equation of the circle of curvature (the osculating circle) of $y = x^3$ at the point $(-1, -1)$.

(3) 10. If $z = z(x, y)$ is defined implicitly by the equation $xy + yz^2 + xz = 3$, what is the value of $\dfrac{\partial z}{\partial x}$ at $(1, 1, 1)$?

(4) 11. Show that if f, g have continuous second derivatives then $z(x, y) = f(x + ay) - g(x - ay)$ is a solution of the wave equation $\dfrac{\partial^2 z}{\partial y^2} = a^2 \dfrac{\partial^2 z}{\partial x^2}$.

(4) 12. Find all local extrema of $f(x, y) = 8x^3 + y^3 - 6x^2 - 6y^2 + 4$.

(5) 13. Find the points on the sphere $x^2 + y^2 + z^2 = 3$ giving the maximum and minimum values of $w = x + 2y + 2z$.

(4) 14. Evaluate:

$$\int_0^2 \int_0^{\sqrt{2y - y^2}} \sqrt{x^2 + y^2}\, dx\, dy$$

(4) 15. Set up a double integral to find the volume of the solid $\mathcal{S}$ enclosed by the surfaces $x = 0$, $y = 2x$, $y = 4$, and $z = 4 - x$.
Set up a triple integral to find the same volume of $\mathcal{S}$ using $dV = dx\, dy\, dz$.
Calculate the volume of $\mathcal{S}$ using one of these integrals.

(4) 16. Sketch (and describe) the solid region whose volume is given by the integral

$$\int_0^{2\pi} \int_{\pi/6}^{\pi/2} \int_0^4 \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

(4) 17. The mass of a solid region $\mathcal{S}$ is given by $\iiint_{\mathcal{S}} \mu(x, y, z)\, dV$, where $\mu(x, y, z)$ is the density function. Use spherical coordinates to find the mass of the solid region $\mathcal{S}$ which lies above $z = \sqrt{x^2 + y^2}$ and below $x^2 + y^2 + z^2 = 9$, given that $\mu(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.