

1. (4 marks) Give the derivative of the function $y = \arctan\left(\frac{1}{1+x}\right)$ and simplify your answer.

2. Evaluate the following integrals.

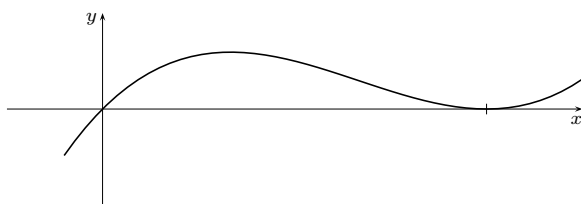
(a) (4 marks) $\int \cos^4(2x) dx$ (b) (4 marks) $\int_2^4 \frac{(\operatorname{arcsec} \sqrt{x})^3}{x \sqrt{x-1}} dx$ (c) (5 marks) $\int x (\ln 5x)^2 dx$

(d) (4 marks) $\int \frac{x^2 dx}{\sqrt{x-4}}$ (e) (4 marks) $\int \sec^4(3x) \tan^4(3x) dx$ (f) (5 marks) $\int \frac{8x^2 + 4x + 5}{(x+1)^2(2x-1)} dx$

(g) (5 marks) $\int_0^{\frac{1}{2}} x \arcsin x dx$ (h) (4 marks) $\int \frac{dx}{\sqrt{4x^2-9}}$

3. (4 marks) Calculate the area of the region bounded by the following: $y = \ln x$; $y = 1$; $x = e^2$

4. (a) (4 marks) Calculate the volume obtained by rotating the region bounded by $y = 0$ and $y = x(x-1)^2$ about the y -axis. (Refer to diagram.)



(b) (2 marks) Set up but *DO NOT EVALUATE* the integral representing the volume obtained by rotating the same region about the x -axis.

5. (8 marks) Evaluate the following limits: (a) $\lim_{x \rightarrow 0^+} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$ (b) $\lim_{x \rightarrow 0^+} (1 + 2 \tan x)^{\frac{1}{x}}$

6. (8 marks) Determine if each of the following improper integrals converges or diverges. If it converges, give its limit.

(a) $\int_{-\infty}^4 \frac{dx}{(4-x)^{2/5}}$ (b) $\int_0^3 \frac{dx}{9-x^2}$

7. (4 marks) Give the particular solution for the differential equation. $\cos^2 x \frac{dy}{dx} = e^{-y} \sin x$; $y(0) = 0$

8. (4 marks) Given the following series, $\sum_{n=1}^{\infty} \frac{9}{(3n-1)(3n+2)}$, let $\{S_n\}$ be the sequence of the partial sums.

(a) Find S_1 , S_2 , S_3 , and S_n . (b) Find the sum of the series.

9. (9 marks) Determine whether the following series converge or diverge. State the test you are using and display a proper solution.

(a) $\sum_{n=1}^{\infty} \frac{\pi + 3 \cos n}{n}$ (b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{4n^2-1}}$ (c) $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$

10. For each alternating series, determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a) (4 marks) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+3\sqrt{n}}$ (b) (3 marks) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n}}{n!}$ (c) (3 marks) $\sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right)^{n^2}$

11. (4 marks) Find the radius of convergence and the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{(-1)^n (x+4)^n}{5^n (2n+1)}$

12. (a) (3 marks) Find the first 4 non-zero terms of the Maclaurin series for $f(x) = \ln(1+x)$

(b) (1 marks) Express the series in \sum form.