CEGEP JOHN ABBOTT COLLEGE

# CALCULUS I (SCIENCE)

(MATHEMATICS 201–NYA–05)



1. Evaluate each of the following limits.

   (a) $\lim\limits_{x\to 2} \dfrac{x^2+2x-8}{x^4-16}$ (b) $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\cos x - \sin x + \sec^2 x}{1-\tan x + x^2}$

   (c) $\lim\limits_{x\to 0-} \left(\dfrac{x}{|x|}+x\right)$ (d) $\lim\limits_{x\to -\infty} \dfrac{9x^4+2x^3-5x+2}{4x^3+6x+1}$

2. Given the function $f(x) = \dfrac{x-1}{x^2+2x-3}$.

   (a) Name the two values of $x$ at which $f$ is not continuous.
   (b) Sketch a graph of $f$.
   (c) List all asymptotes and removable discontinuities of $f$.

3. Find all values of $c$ which make $f(x) = \begin{cases} 3cx+1 & \text{if } x \leqslant -2, \\ 2c - \frac{1}{2}x^2 & \text{if } x > -2, \end{cases}$ continuous at $x = -2$.

4. Use the *definition* of the derivative to find $f'(x)$, where $f(x) = \sqrt{5-3x}$.

5. Differentiate each of the following. *Do not simplify your answers*.

   (a) $y = 3^x - x^3 - \dfrac{3}{x} - \dfrac{x}{3} + e^3$ (b) $y = \cos^3(\sqrt{x+1}\,)$

   (c) $r = \dfrac{2t}{\sqrt{2t^2-t+3}}$ (d) $y = \ln\left(\dfrac{(2-x)^5\sqrt{x+1}}{(x+3)^7}\right)$

   (e) $z = (\sin x)^x$ (f) $y = e^{\tan x} + \tan(e^x)$

6. Differentiate $y = \left(\dfrac{ax+b}{cx+d}\right)^5$ and *simplify your answer*.

7. Find all critical numbers of $f(x) = (x^2-9)^3(3x+5)^2$.

8. Sketch the graph of a function with these characteristics: $f(2) = f(4) = 0$; $f'(x) > 0$ if $x < 3$, $f'(3)$ is undefined and $f'(x) < 0$ if $x > 3$; $f''(x) > 0$ if $x \neq 3$.

9. Find the absolute extrema of $f(x) = x^4 - 8x^2 + 7$ on $[-3, 1]$.

10. Given $x^{2/3} + y^{2/3} = 2$:

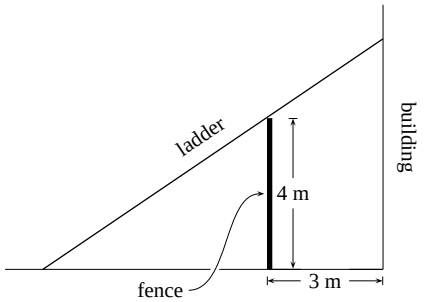
    (a) Find $\frac{dy}{dx}$. (b) Find all points where the slope of the tangent line is $-1$.
    (c) Find an equation of the tangent line at the point $(-1, 1)$. (d) Find $\frac{d^2y}{dx^2}$.

11. If the graphs of $f$ and $g$ both pass through the origin, where each of $f$ and $g$ is differentiable, at what slope does the graph of the product $fg$ pass through the origin? Justify your answer.

12. Let $s(t) = \sin 2t - t$ be the equation of motion of a body, for $0 \leqslant t < \frac{1}{2}\pi$. Find the acceleration of the body when its velocity is zero.

13. Two cars start moving from the same point at the same time: one travels south at 100 km/h and the other travels west at 40 km/h. How fast is the distance between them increasing 90 minutes later?

14. A 4 m tall fence is parallel to a tall building, 3 m away from the building. How long is the shortest ladder which, starting on the opposite side of the fence from the building, will reach from the ground over the fence, to touch the wall?




15. Sketch the graph of $f(x) = x\sqrt{8-x^2}$, given

$$f'(x) = \frac{8-2x^2}{\sqrt{8-x^2}} \quad \text{and} \quad f''(x) = \frac{2x^3-24x}{(8-x^2)^{3/2}}.$$

    Be sure to indicate the domain of $f$, all intercepts, extrema, inflection points, intervals of increase/decrease, and intervals of concavity.

16. Suppose that $f$ and $g$ are functions such that:

    - $f$ has a continuous second derivative on $\mathbb{R}$;
    - $f(0) = 2$, $f'(0) = -3$ and $f''(0) = 0$;
    - $g'(x) = e^{-2x}(2f'(x) + 3f(x))$, for $x \in \mathbb{R}$.

    (a) Is there sufficient information to determine whether the graph of $f$ has an inflection point at $x = 0$? Justify your answer.
    (b) Show that $g''(x) = e^{-2x}(2f''(x) - f'(x) - 6f(x))$.
    (c) Does $g$ have a local maximum at $x = 0$? Justify your answer.

17. Evaluate each of the following integrals.

    (a) $\displaystyle\int \frac{1-x+x^2}{x\sqrt{x}}\,dx$ (b) $\displaystyle\int (x^e + e^x - \sin x)\,dx$

    (c) $\displaystyle\int_{1/e}^{e} \left(e^x - \frac{1}{x}\right) dx$ (d) $\displaystyle\int_{\pi/6}^{\pi/3} \csc x(\csc x - \cot x)\,dx$

18. (a) Evaluate $\displaystyle\int_{\frac{1}{9}\pi}^{\frac{1}{9}\pi} x^2 \tan x\,dx$. (b) Find $F'(2)$ if $F(x) = \displaystyle\int_0^x \frac{dz}{1+z+z^2}$.

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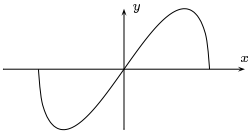
## ANSWERS

1. (a) $\frac{3}{16}$ (b) $32\pi^{-2}$ (c) $-1$ (d) $-\infty$
2. (a) $-3, 1$ (b) The graph of $f$ is a rectangular hyperbola (with vertical asymptote $x = -3$ and horizontal asymptote $y = 0$) less the point $(1, \frac{1}{4})$. (c) $f$ has a removable discontinuity at $x = 1$ and an infinite discontinuity (and therefore a vertical asymptote) at $x = -3$.
3. $c = \frac{3}{8}$ 4. $f'(x) = \displaystyle\lim_{t\to x} \frac{\sqrt{5-3t}-\sqrt{5-3x}}{t-x} = \frac{-3}{2\sqrt{5-3x}}$
5. (a) $3^x \ln 3 - 3x^2 + 3x^{-2} - \frac{1}{3}$ (b) $-\dfrac{\cos^2\sqrt{x+1}\,\sin\sqrt{x+1}}{2\sqrt{x+1}}$

   (c) $\dfrac{2}{\sqrt{2t^2-t+3}} - \dfrac{t(4t-1)}{(2t^2-t+3)^{3/2}}$ (d) $\dfrac{5}{x-2} + \dfrac{1}{2(x+1)} - \dfrac{7}{x+3}$

   (e) $(\sin x)^x(\ln\sin x + x\cot x)$ (f) $e^{\tan x}\sec^2 x + e^x\sec^2(e^x)$
6. $5(ad-bc)(ax+b)^4(cx+d)^{-6}$ 7. $\pm 3, -\frac{5}{3}, = \frac{9}{4}$ and 1.
8. The graph of $1-(x-3)^{2/3}$ will do, for example.
9. Max: $f(-3) = 16$; min: $f(-2) = -9$.
10. (a) $-\sqrt[3]{(y/x)}$ (b) $(\pm 1, \pm 1)$ (c) $x - y + 2 = 0$ (d) $\frac{2}{3}(x^4y)^{-1/3}$
11. $(fg)'(0) = f'(0)g(0) + f(0)g'(0) = 0$ $(\because f(0) = g(0) = 0)$, so the graph of $fg$ passes through the origin with slope 0.
12. $a = -2\sqrt{3}$ when $v = 0$, (*i.e.*, when $t = \frac{1}{6}\pi$). 13. $20\sqrt{29}$ km/hr.
14. $\left(3^{2/3} + 4^{2/3}\right)^{3/2}$ m
15. 


    Domain: $[-2\sqrt{2}, 2\sqrt{2}\,]$.
    Intercepts: $(0, 0)$, $(\pm 2\sqrt{2}, 0)$.
    Extrema: $(-2, -4)$ (min); $(2, 4)$ (max).
    IP: $(0, 0)$
16. (a) No, because $f''$ might not change sign at $x = 0$. (b) Use the product rule. (c) $g'(0) = 0$, $g''$ is continuous near $x = 0$ and $g''(0) = -9 < 0$, so $g$ has a local maximum at $x = 0$ by the second derivative test.
17. (a) $-2x^{-1/2} - 2x^{1/2} + \frac{2}{3}x^{3/2} + C$ (b) $x^{e+1}(e+1)^{-1} + e^x + \cos x + C$
    (c) $e^e - e^{1/e} - 2$ (d) $\frac{4}{3}\sqrt{3} - 2$
18. (a) 0 (b) $\frac{1}{7}$