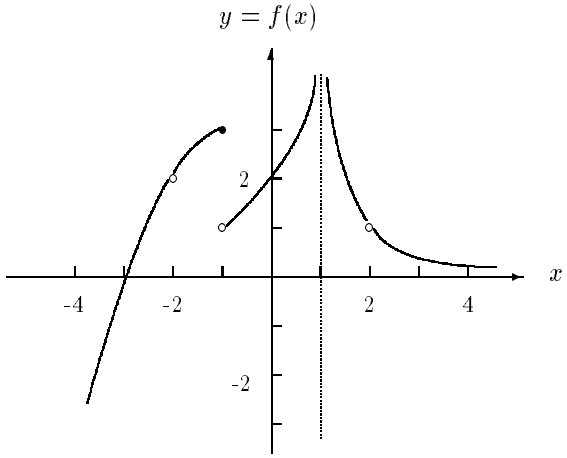
(Marks)

(7) 1. By referring to the graph of $f(x)$, determine the following limits:



(a) $\lim_{x\to -3} f(x) =$ (b) $\lim_{x\to -1^+} f(x) =$
(c) $\lim_{x\to -1} f(x) =$ (d) $\lim_{x\to -2} f(x) =$
(e) $\lim_{x\to 2^-} f(x) =$ (f) $\lim_{x\to 1^-} f(x) =$
(g) $\lim_{x\to -\infty} f(x) =$

(5×2) 2. Evaluate the following limits. Note that the limit may be ∞ or $-\infty$. State *dne* (does not exist) if the limit is neither $\pm\infty$ nor a finite real number.

(a) $\lim_{x\to \sqrt{5}^-} \dfrac{12x-5}{x^2-5}$ (b) $\lim_{x\to e} \left(\pi^{x-e} + 2\ln x\right)$

(c) $\lim_{x\to 3} \dfrac{x^2+x-12}{2x^2-5x-3}$ (d) $\lim_{x\to 2} \dfrac{\frac{1}{x+1} - \frac{1}{3}}{x^2-4}$

(e) $\lim_{x\to -\infty} \dfrac{5x^3+2}{x^2-3}$

(1) 3. (a) State the definition of the derivative, as a limit.

(3) (b) Use the definition from part (a) to find the derivative of $f(x) = \sqrt{x-5}$.

(1) 4. (a) State the three conditions for a function $f(x)$ to be continuous at $x = c$.

(3) (b) State any discontinuities for the following function. *Justify your answer.*

$$f(x) = \begin{cases} \dfrac{1}{x+1} & x < 0 \\ x^2 & 0 \le x \le 2 \\ 4-x & x > 2 \end{cases}$$

(6×4) 5. Find $\dfrac{dy}{dx}$ or y' for each of the following. *It is not necessary to simplify your answers.*

(a) $y = 4x^3 - \dfrac{3}{x^4} + \dfrac{1}{\sqrt[3]{x^4}} + 3^4$ (b) $y = \dfrac{e^x+1}{e^{2x}+2}$

(c) $y = 4^x \ln(x^2-4)$ (d) $y = (1+x^2)^{\sqrt{x}}$

(e) $y = \dfrac{x^4\sqrt[5]{1+x^2}}{\sin^3 x}$ (f) $y = \tan^4\left(e^{-3x^2}\right)$

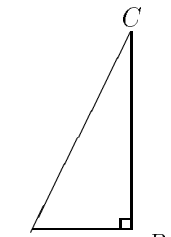
6. Use implicit differentiation to find the slope of the tangent line to the curve $x^2 + 4y^2 - 7x + 6 = 0$ at the point $(2,-1)$. (4)

7. The equation of motion of a particle moving along a straight line is $s = t^3 - 4t^2 + 4t + 11$ where s is measured in meters and t in seconds. (4)

(a) Find the velocity at time t.

(b) When is the particle at rest?

8. Let ABC be a right triangle whose base AB is 7 cm long. The height BC is increasing in length at a rate of 2 cm/sec. When $BC = 24$ cm, at what rate is the length of the hypotenuse changing? (4)



9. Find the absolute maximum and the absolute minimum of $f(x) = x^3 + 3x^2 - 24x + 3$ on the interval $[0,4]$. (4)

10. Given $f(x) = 12 + 2x^2 - x^4$ (8)

(a) Determine all regions where $f(x)$ is increasing, decreasing, concave up and concave down.

(b) Sketch the graph of $f(x)$. Show all asymptotes (with equations), and relative extrema and points of inflection (with coordinates), *if any.*

11. Find the point on the graph of $2y = x^2$, closest to the point $(32,1)$. (5)

12. Determine the area under the graph of $y = x + 4\cos x$ between $x = 0$ and $x = \pi/2$. (3)

13. Evaluate $\displaystyle\int_0^4 \sqrt{16-x^2}\,dx$ by considering the area under the graph of a function. *Your answer should include the appropriate picture.* (3)

14. Find $S(t)$ given $S''(t) = 30\sqrt{t}$, $S(4) = 200$, and $S'(1) = 10$. (4)

15. Evaluate the following integrals: (4×3)

(a) $\displaystyle\int \left(4x^3 - \frac{4}{x^3} + \sqrt[4]{x^3} - \pi e^x + \ln\pi\right) dx$

(b) $\displaystyle\int \frac{(x+2)^2}{x^2}\,dx$ (c) $\displaystyle\int_{-1}^{1} (x^2 - e^x)\,dx$

(d) $\displaystyle\int_1^4 \frac{2x+3}{\sqrt{x}}\,dx$

(Marks)

ANSWERS

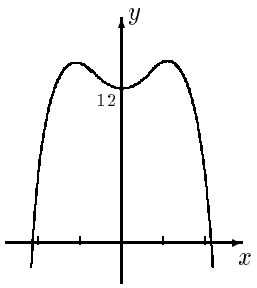
1. 0, 1, DNE, 2, 1, ∞, $-\infty$
2. $-\infty$, 3, 1, $-1/36$, $-\infty$
3. (b) $f'(x) = \lim_{h\to 0} \dfrac{\sqrt{x+h-5} - \sqrt{x-5}}{h} = \cdots = \dfrac{1}{2\sqrt{x-5}}$
4. (b) Discontinuous at $x = -1,\ 0,\ \&\ 2$
5. (a) $12x^2 + 12x^{-5} - \dfrac{4}{3}x^{-7/3}$

   (b) $\dfrac{\mathrm{e}^x(\mathrm{e}^{2x}+2) - 2\mathrm{e}^{2x}(\mathrm{e}^x+1)}{(\mathrm{e}^{2x}+2)^2}$

   (c) $4^x(\ln 4)\ln(x^2-4) + 4^x\left(\dfrac{2x}{x^2-4}\right)$

   (d) $(1+x^2)^{\sqrt{x}}\left(\dfrac{1}{2\sqrt{x}}\ln(1+x^2) + \dfrac{2x\sqrt{x}}{1+x^2}\right)$

   (e) $\dfrac{x^4\sqrt[5]{1+x^2}}{\sin^3 x}\left(\dfrac{4}{x} + \dfrac{2x}{5(1+x^2)} - \dfrac{3\cos x}{\sin x}\right)$

   (f) $4\tan^3(\mathrm{e}^{-3x^2})\sec^2(\mathrm{e}^{-3x^2})(-6x\mathrm{e}^{-3x^2})$
6. $-3/8$
7. (a) $v(t) = 3t^2 - 8t + 4$, (b) $t = 2/3$, 2 s.
8. 1.92 cm/sec.
9. MIN $(2, -25)$ MAX $(4, 19)$
10. 

    Maxima $(\pm 1, 13)$, Min $(0, 12)$
    Inflection Points $(\pm 1/\sqrt{3}, 113/9)$
    (No asymptotes).
11. $(4, 8)$
12. $\dfrac{\pi^2 + 32}{8}$
13. 4π
14. $S(t) = 8t^{5/2} - 10t - 16$
15. (a) $x^4 + 2x^{-2} + \dfrac{4}{7}x^{7/4} - \pi\mathrm{e}^x + x\ln\pi + C$

    (b) $x + 4\ln|x| - \dfrac{4}{x} + C$

    (c) $\dfrac{2}{3} - \mathrm{e} + \dfrac{1}{\mathrm{e}}$

    (d) $46/3$