Final Exam
201-DDB Calculus
December 2004

1. (a) Use the Binomial Series to find the first four terms of the Maclaurin series for $f(x) = \dfrac{1}{\sqrt{1+x^3}}$

   (b) Approximate $\displaystyle\int_0^{0.5} \frac{1}{\sqrt{1+x^3}}\,dx$ correct to four decimal places.

2. Given $f(x) = \displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{(n+1)2^n}$

   (a) Find the domain of f

   (b) Find$f^{(5)}(3)$ without differentiating 5 times.

3. (a) Find a Taylor polynomial of degree 3 *i.e.* $p_3(x)$ for $f(x) = x \ln x$ centered at $c = 1$ and an expression for the Remainder term $R_3(x)$.

   (b) Estimate the error in using $p_3(x)$ to approximate $f(x)$ on the interval $[0.5, 1.5]$

4. (a) Use multiplication of power series to find the first five nonzero terms of the Maclaurin series for $f(x) = \dfrac{\cos x}{1+x}$ and state its radius of convergence

   (b) Find the Taylor series for $f(x) = \dfrac{3}{4-x}$ about $c = -2$ and state its radius of convergence.

5. Given the curve C having parametric equations $x = t^2$, $y = t^3 - 4t$

   (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

   (b) Find all points on C where the tangent line is vertical or horizontal.

   (c) Sketch the graph of C showing the orientation of the curve.

6. (a) Sketch $r^2 = 8\sin 2\theta$ and $r = 2$ on the same set of axes.

   (b) Find all points of intersection.

   (c) **Set up** the integral needed to find the area of the region inside $r^2 = 8\sin 2\theta$ but outside $r = 2$

7. (a) Sketch the space curve defined by $\vec{r}(t) = (2\cos t\,,\, 3t\,,\, 2\sin t)$

   (b) Find:

      i. the velocity, the acceleration and the speed of a particle moving along this curve.

      ii. the unit tangent and the principal unit normal vectors $\vec{T}$ and $\vec{N}$

      iii. the equation of the tangent line to the curve at $t = \pi/3$

      iv. the curvature at $t = \pi/3$

8. (a) Given $z = e^{3xy^2} + 4x^3 - y^3 \ln x$, find $\frac{\partial^2 z}{\partial y \partial x}$

   (b) Given that $z = f(x, y)$ is implicitly defined by the equation $xy^2z + 4x^3 - \sqrt{x}\,\sin z + y\,e^{x^2} = 0$, find $\frac{\partial z}{\partial y}$

9. Given $z = f(x, y)$; $x = r\cos\theta$; $y = r\sin\theta$

   (a) Find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$

   (b) Show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

10. Let $f(x, y, z) = 2x^2 + y^2 - z$. Show that the line whose parametric equations are $\{ x = -2+t,\ y = t,\ z = -4+4t \}$ is tangent to the surface $f(x, y, z) = 0$ at a certain point P. Find the equation of the tangent plane at P.

11. Find and classify the critical points of $f(x, y) = 16xy - x^4 - 2y^2$

12. Use the Method of Lagrange Multipliers to find the point on the plane $2x - y + z = 1$ closest to the point $P(-4, 1, 3)$.

13. Sketch and describe (or give the name) of the following surfaces:

    (a) $4x^2 - 4y + z^2 = 0$

    (b) $z = \sqrt{r^2 - 1}$

    (c) $\rho = \sin\theta \sin\varphi$

14. Evaluate $\displaystyle\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{1+y^4}\, dy\, dx$ by reversing the order of integration.

15. Combine the sum of the double integrals:

$$\int_0^2 \int_0^x \sqrt{x^2+y^2}\, dy\, dx + \int_2^{2\sqrt{2}} \int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$

    into one double integral by converting to polar coordinates.

16. Find the volume of the solid region bounded above by the the sphere $x^2 + y^2 + z^2 = 6$ and below by the paraboloid $z = x^2 + y^2$ using a triple integral in cylindrical coordinates.