CEGEP JOHN ABBOTT COLLEGE

CALCULUS II (SCIENCE)
(MATHEMATICS 201-NYB/∞)

1. Differentiate $y = \arctan e^x + \operatorname{arcsec} x^2$

2. Evaluate $\displaystyle\int \frac{x \arcsin x^2}{\sqrt{1-x^4}}\,dx$

3. Evaluate $\displaystyle\int \frac{dx}{x\sqrt{9+16x^2}}$

4. Evaluate $\displaystyle\int_2^5 \frac{x^2}{\sqrt{x-1}}\,dx$

5. Evaluate $\displaystyle\int \csc^4 x\sqrt{\cot x}\,dx$

6. Evaluate $\displaystyle\int_e^{e^2} (\ln x)^2\,dx$

7. Evaluate $\displaystyle\int \sin^2 x + \cos^3 2x\,dx$

8. Evaluate $\displaystyle\int x^2 e^{3x}\,dx$

9. Find the area of the region bound by $y = x^3$ and $y = \dfrac{32}{x^2}$ between $x = 1$ and $x = 2$.

10. Set up, *but do not evaluate*, an integral that yields the volume of the solid obtained by revolving the region bound by $y = 1 + \cos x$, and $y = 1$ from $x = 0$ to $x = \frac{\pi}{2}$
    (a) about the x-axis, (b) about the y-axis.

11. Evaluate each limit.
    (a) $\displaystyle\lim_{x\to\frac{\pi}{4}} \frac{1-\tan x}{4x-\pi}$ (b) $\displaystyle\lim_{x\to 0} (\cos x)^{1/x^2}$

12. Determine whether each improper integral converges or diverges. If it converges give its limit.
    (a) $\displaystyle\int_1^\infty \frac{3}{(x+5)(x+2)}\,dx$ (b) $\displaystyle\int_0^4 \frac{1}{(x-2)^{4/3}}\,dx$

13. Solve the differential equation $\cos^2 x\,\dfrac{dy}{dx} = \sec 2y$ for the initial condition where $x = \frac{\pi}{4}$ and $y = \frac{\pi}{6}$.

14. Determine the whether the sequence converges or diverges. If the sequence converges give its limit.
$$\left\{ n \arcsin \frac{1}{n} \right\}.$$

15. Find the sum of the series $\displaystyle\sum_{n=1}^\infty \frac{3^n - 2^n}{4^n}$

16. Determine whether each series converges or diverges.
    (a) $\displaystyle\sum_{n=2}^\infty \frac{3}{n\sqrt{\ln n}}$ (b) $\displaystyle\sum_{n=1}^\infty \sqrt{\frac{2n+1}{3n-2}}$ (c) $\displaystyle\sum_{n=3}^\infty \left(\frac{\ln n}{n}\right)^n$

17. For each alternating series, determine whether the series is absolutely convergent, conditionally convergent, or divergent.
    (a) $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n (n+2)!}{e^n}$ (b) $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n}{1+\sqrt[3]{n^2}}$

18. Determine the interval of convergence of the power series
$$\sum_{n=1}^\infty \frac{2^n (x-3)^n}{n^2}.$$

19. Find the first four terms of the Maclaurin series of $f(x) = e^{-3x}$.

ANSWERS

1. $\dfrac{e^x}{1+e^{2x}} + \dfrac{2}{x\sqrt{x^4-1}}$

2. $\frac{1}{4}\arcsin^2 x^2 + C$

3. $\frac{1}{3}\ln\left|\dfrac{\sqrt{9+16x^2}-3}{x}\right| + C$

4. 356/15

5. $-\frac{2}{3}\cot^{3/2} x - \frac{2}{7}\cot^{7/2} x + C$

6. $e(2e-1)$

7. $\frac{1}{2}x + \frac{1}{4}\sin 2x - \frac{1}{6}\sin^3 2x + C$

8. $\frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$

9. $49/4$

10. (a) $\displaystyle\pi\int_0^{\frac{\pi}{2}} \left\{(1+\cos x)^2 - 1\right\}\,dx$ (b) $\displaystyle 2\pi\int_0^{\frac{\pi}{2}} x\cos x\,dx$

11. (a) $-1/2$ (b) $1/\sqrt{e}$

12. (a) The integral converges to $\ln 2$.
    (b) The integral diverges (to ∞).

13. $2\sin 2y = 4\tan x + \sqrt{3} - 4$

14. The sequence converges to 1.

15. The series converges to 2.

16. (a) The series diverges by the integral test.
    (b) The series diverges by the vanishing criterion ($a_n \to \sqrt{2/3}$).
    (c) The series converges by the root test ($\sqrt[n]{a_n} = (\ln n)/n \to 0$).

17. (a) Since $(n+2)! > e^n$ if $n \geqslant 1$, the series diverges by the vanishing criterion.
    (b) The series is conditionally convergent by the AST, and the LCT with $\sum n^{-2/3}$.

18. The interval of convergence of the series is $\left[\frac{5}{2}, \frac{7}{2}\right]$.

19. $T_3(x) = 1 - 3x + \frac{9}{2}x^2 - \frac{9}{2}x^3$