NYB **FINAL EXAM** **FALL 2002**

1. Evaluate the following:

   (a) $\displaystyle\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

   (b) $\displaystyle\int_0^4 (t-1)\sqrt{1+2t}\ dt$

   (c) $\displaystyle\int (1+\sin x)^2\ \ dx$

   (d) $\displaystyle\int \tan^3\left(\frac{x}{2}\right)\sec^4\left(\frac{x}{2}\right) dx$

   (e) $\displaystyle\int \frac{\sqrt{4x^2-9}}{x}\ dx$

   (f) $\displaystyle\int \sqrt{x}\ln x\, dx$

   (g) $\displaystyle\int \frac{8x^3-6x^2+3x-4}{x^2\left(x^2+1\right)}\ dx$

2. Calculate the folowing limits:

   (a) $\displaystyle\lim_{x\to 0}\frac{x-\sin x}{x-\tan x}$

   (b) $\displaystyle\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$

3. Determine whether the following integrals converge or diverge. If an integral converges, give the exact value.

   (a) $\displaystyle\int_0^\infty xe^{x^2}\, dx$

   (b) $\displaystyle\int_0^9 \frac{\sqrt{x}+1}{\sqrt{x}} dx$

4. Find the solution, y, for $\left(x^2+1\right)y' = xy$ when $y>0$ and $y\left(0\right)=1$.

5. $\Re$ is the region bounded by $y=\dfrac{8}{x}$ and $y=6-x$

   (a) Find the exact value of the area for the region $\Re$.

   (b) Find the volume of the solid of revolution when $\Re$ is rotated about the Y-axis.

   (c) Set up the definite integral that represents the volume of the solid of revolution when $\Re$ is rotated about the X-axis.

6. Consider the sequence $\{a_n\} = \left\{\dfrac{3n-1}{4n+3}\right\}$

   (a) Does the sequence converge, and if so, to what value?

(b) Does the corresponding series $\sum_{n=1}^{\infty} a_n$ converge? **Justify your answer**.

7. Given the series $\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{7^n}$

   (a) Find the formula for S_n, the n^{th} partial sum of the series.

   (b) Use the formula in part (a) to find the sum of the series.

8. Determine whether the following series converge or diverge. State the test you are using and display a proper solution.

   (a) $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

   (b) $\sum_{n=0}^{\infty} \frac{n^n}{2^n\, n!}$

   (c) $\sum_{n=1}^{\infty} \frac{\cos n + 2}{\sqrt{3n+1}}$

   (d) $\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^2}$

9. Determine whether the following series are absolutely convergent, conditionally convergent or divergent:

   (a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{5^{n+1}\,(n^3+1)}{7^n}$

   (b) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{7n+2}{3n-1}\right)^n$

   (c) $\sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{n}}{n+20}$

10. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^{n+1}\,(x+1)^n}{5^n\sqrt{3n+1}}$

11. Given $f(x) = \ln x$

   (a) Find the first 3 non-zero terms of the Taylor's series expansion of $f(x)$ centered at $a = 1$.

   (b) Use sigma notation to write the general form of the series in (a).

Answers.

1 $(a)\ \dfrac{\pi^2}{32}$ $(b)\ \dfrac{56}{5}$ $(c)\ \dfrac{3}{2}x - 2\cos x - \dfrac{1}{4}\sin(2x) + C$ $(d)\ \dfrac{1}{2}\tan^4\left(\dfrac{x}{2}\right) + \dfrac{1}{3}\tan^6\left(\dfrac{x}{2}\right) + C$

$(e)\ \sqrt{4x^2-9} - 3\sec^{-1}\left(\dfrac{2x}{3}\right) + C$ $(f)\ \dfrac{2}{3}x^{3/2}\ln x - \dfrac{4}{9}x^{3/2} + C$

$(g)\ 3\ln|x| + \dfrac{4}{x} + \dfrac{5}{2}\ln\left(x^2+1\right) - 2\tan^{-1}x + C$

2 $(a)\ \dfrac{-1}{2}$ $(b)\ e^6$

3 (a) div (b) conv

4 $y = \sqrt{x^2+1}$

5 $(a)\ 6 - 8\ln 2$ $(b)\ \dfrac{8\pi}{3}$ $(c)\ V = \pi\displaystyle\int_2^4 \left((6-x)^2 - \left(\dfrac{8}{x}\right)^2\right)dx$

6 (a) conv (b) div by the n^{th} term test

7 $(a)\ s_n = \dfrac{a}{1-r}(1-r^n) = \dfrac{\frac{4}{7}}{1+\frac{2}{7}}\left(1 - \left(\dfrac{-2}{7}\right)^n\right)$

$(b)\ \displaystyle\lim_{n\to\infty} s_n = \dfrac{4}{9}$

8 (a) conv by LCT with $b_n = \dfrac{1}{n^2}$

(b) div by ratio test with lim$= \dfrac{1}{2}e > 1$

(c) div by CT with $b_n = \dfrac{1}{\sqrt{3n+1}}$ which div by LCT or Integral Test

(d) conv by Integral Test with $\displaystyle\int_2^\infty \dfrac{1}{n(\ln n)^2}dn$ conv to $\dfrac{1}{\ln 2}$

9 (a) AC by ratio test (b) div by the root test (c)CC

10 IOC$= [\dfrac{-7}{2}, \dfrac{3}{2})$ and ROC $= \dfrac{5}{2}$

11 $(a)\ (x-1) - \dfrac{1}{2}(x-1)^2 + \dfrac{1}{3}(x-1)^3 - \cdots$ $(b)\ \displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\dfrac{(x-1)^n}{n}$