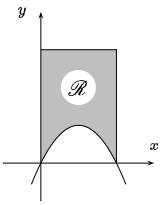
CEGEP JOHN ABBOTT COLLEGE

CALCULUS II (SCIENCE)

(MATHEMATICS 201–NYB/∞)

1. Let $\mathscr{R}$ be the region in quadrant I bounded by $y = 2x - x^2, \quad y = 3, \quad x = 0$ and $x = 2$.



   (a) Find the area of the region $\mathscr{R}$.

   (b) $\mathscr{R}$ is revolved about the y-axis. Set up the integral to find the volume of th solid generated. *Do not evaluate the integral.*

   (c) $\mathscr{R}$ is now revolved about the x-axis. Set up the integral to find the volume of the solid generated. *Do not evaluate the integral.*

2. Evaluate the following integrals.

   (a) $\displaystyle\int e^{2x} \sin x \, dx$ (b) $\displaystyle\int_0^{\frac{\pi}{3}} (\sin^3 \vartheta + \cos^2 \vartheta) \, d\vartheta$

   (c) $\displaystyle\int \frac{dx}{x^2\sqrt{4x^2 - 9}}$ (d) $\displaystyle\int \frac{12x^2 + 7x + 1}{(x-1)(x+1)^2} \, dx$

   (e) $\displaystyle\int_3^7 \frac{x \, dx}{(2x-5)^{3/2}}$ (f) $\displaystyle\int \arcsin x \, dx$

3. Find *and simplify* the derivative of $f(x) = \dfrac{\arctan x^2}{1 + x^4}$.

4. Evaluate the following limits.

   (a) $\displaystyle\lim_{x \to 0} \frac{x \cos x - \sin x}{\tan x - x}$ (b) $\displaystyle\lim_{x \to \frac{\pi}{2}^-} \sin x^{\sec x}$

   (c) $\displaystyle\lim_{x \to 0} \left(\frac{1}{e^t - 1} - \frac{1}{t} \right)$

5. Determine whether each of the following integrals converges or diverges. If an integral converges, give its exact value.

   (a) $\displaystyle\int_0^{\infty} \frac{x \, dx}{x^2 + 1}$ (b) $\displaystyle\int_{\frac{1}{2}}^2 \frac{dx}{\sqrt[3]{2x-1}}$

6. Solve the differential equation: $(1 + x^2)y' = y - 1; \;\; y(0) = 2$.

7. Determine whether each sequence is convergent or divergent. If convergent, give the limit. (Jusify your answer.)

   (a) $a_n = \dfrac{n!}{(n+2)!}$ (b) $a_n = \cos \dfrac{n\pi}{2}$

8. Calculate, if possible, the sum of the series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+1)}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{5^{n-1}}$

9. Classify each of the following series as convergent or divergent. (Briefly justify your conclusions.)

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^3 + 2n + 5}}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2 + 5}{4n^2 - 1}$ (d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{2^n} - \frac{2}{3n} \right)$

10. Determine whether the following series are abolutely convergent, conditionally convergent or divergent. (Briefly justify your conclusions.)

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

11. Find the radius and interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{(n+1)3^n}$$

12. Find the first four terms in the Maclaurin series for

$$f(x) = \sqrt{x+4}.$$

ANSWERS

1. (a) $A = \frac{14}{3}$
   (b) $V = 2\pi \int_0^2 x(3 - 2x + x^2) \, dx$
   (c) $V = \pi \int_0^2 9 - (2x - x^2)^2 \, dx$
   (d) $\ln\left|(x-1)^5(x+1)^7\right| + \frac{3}{x+1} + C$

2. (a) $\frac{e^{2x}}{5}(2\sin x - \cos x) + C$
   (b) $\frac{4\pi+3\sqrt{3}+5}{24}$
   (c) $\frac{\sqrt{4x^2-9}}{9x} + C$

3. $f'(x) = \dfrac{2x(1 - 2x^2 \arctan x^2)}{(1+x^4)^2}$
   (e) $\frac{8}{3}$
   (f) $x \arcsin x - \sqrt{1-x^2} + C$

4. (a) -1 (b) 1 (c) $-\frac{1}{2}$

5. (a) The integral diverges.
   (b) The integral converges to $\frac{3}{4}\sqrt[3]{9}$.

6. $y = 1 + e^{\arctan x}$

7. (a) 0 (b) divergent (oscillating)

8. (a) $\frac{1}{2}$ (b) $\frac{10}{3}$

9. (a) Divergent (lim. comp. with $1/\sqrt{n}$)
   (b) Convergent ($\int$-test)
   (c) Divergent (the terms approach $3/4$)
   (d) Divergent (lim. comp. with $1/n$)

10. (a) Abs. conv. (RaT, $\left|\frac{a_{n+1}}{a_n}\right| \to 1/e$)
    (b) Cond. conv. (AST &, *e.g.*, comp. with $1/n$)

11. $R = 3 \qquad I = [-2, 4)$

12. $2 + \frac{x}{4} - \frac{x^2}{64} + \frac{x^3}{512}$