CEGEP
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COLLEGE

CALCULUS I (SCIENCE)

(MATHEMATICS 201-NYA/∞)

1. Evaluate each limit using algebraic techniques. If a limit does not exist, assign one of the symbols ∞ or $-\infty$ if possible.

   (a) $\displaystyle\lim_{x\to 3}\frac{x^2+2x-15}{3(2x-1)(x-3)}$ (b) $\displaystyle\lim_{x\to\frac{\pi}{4}}\left\{\sin x\cos x-\tan\left(\frac{\pi}{2}-x\right)\right\}$

   (c) $\displaystyle\lim_{x\to-\infty}\frac{2x^4-x^2+2x-1}{x^3+3x^2-x+3}$ (d) $\displaystyle\lim_{x\to 2^-}\frac{-4}{(x-2)^3}$

   (e) $\displaystyle\lim_{x\to 1}\frac{x-1}{\sqrt{x^2+3x}-2}$

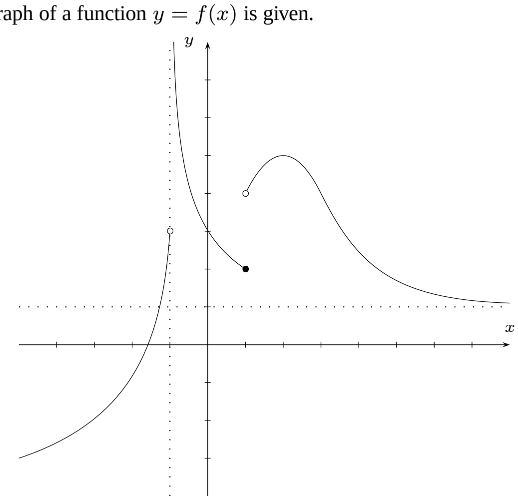
2. Given $f(x)=\dfrac{\sin x}{x(x+1)}$.

   (a) Complete the table below:

x (radians)	0.1	0.01	0.001	0.0001
$f(x)$				

   (b) Use the above table to predict $\displaystyle\lim_{x\to 0^+}\frac{\sin x}{x(x+1)}$.

3. The graph of a function $y=f(x)$ is given.



   (a) Find:

   (i) $\displaystyle\lim_{x\to-1^-}f(x)$ (ii) $\displaystyle\lim_{x\to 1^+}f(x)$ (iii) $\displaystyle\lim_{x\to 1}f(x)$

   (iv) $\displaystyle\lim_{x\to+\infty}f(x)$ (v) $f(1)$

   (b) f is *not continous* at $x=1$. State which property of continuity is not satisfied.

4. If $f(x)=\begin{cases}4k-x^2 & \text{if } x<-1\\ 2-kx & \text{if } x\geqslant -1\end{cases}$ where k is a non-zero constant, find a value (or values) of k which would make f continous at $x=-1$.

5. For $f(x)=2+\dfrac{1}{x-1}$, use the limit definition of the derivative to find $f'(x)$.

6. Differentiate each function with respect to x. *Do not simplify your answers.*

   (a) $y=2^x-x^2+\dfrac{2}{x}+2\sqrt{x}+\sqrt{2}$ (b) $f(x)=(\tan x-\cos x)^{6/5}$

   (c) $g(x)=(1+\sec 3x)\,e^{5-x^2}$ (d) $h(x)=\dfrac{3x-5e^x}{x^2-6x+2}$

   (e) $y=(\sin x)^{x^2}$

7. If $y=\dfrac{\sqrt[3]{(4-5x)^2}}{(x+1)^4(x^2+2)}$, use logarithmic differentiation to find $\dfrac{dy}{dx}$ as a function of x.

8. If $f(x)=\sin x\tan x$

   (a) find $f''(x)$, and
   (b) evaluate $f''(0)$.

9. (a) If $f(x)=x^2+1$ sketch the *derivative* function $f'(x)$.

   (b) Sketch a function defined by $y=h(x)$ whose instantaneous rate of change is -2.

   (c) The slope of the tangent line to a function $y=f(x)$ at any point (x,y) on the curve is $m(x)=\cos x$. If the curve passes through $(0,0)$,

   (i) find $y=f(x)$,
   (ii) find an equation of the tangent line to the curve at the point $(0,0)$.

10. If $\sqrt{x}y+x^2y^3=1+3y$,

    (a) use implicit differentiation to find $\dfrac{dy}{dx}$,
    (b) find the slope of the tangent line at the point $(1,-1)$ on the curve.

11. A box is to be made from a square piece of cardboard with a 60 cm side by cutting out equal squares from the corners, then turning up the sides. What size square should be cut out so that the box has maximum volume? What is the volume of the box?

12. Coffee is leaking out of a hemispheric bowl at the rate of 5 ml per hour; the radius of the bowl is 6 cm. Find the rate at which the coffee level is falling when the depth of the coffee is only 2 cm. (For a bowl of radius 6 cm, the volume of a spherical sector is $V=\pi h^2(6-h/3)$. Note that 1 ml $=1$ cm^3.)

13. Without graphing, find the absolute maximum and absolute minimum values of $f(x)=\left(x^2-1\right)^{4/3}$ on the interval $[0,3]$. Show all supporting work.

14. Given

$$f(x)=\frac{x}{(x-2)^2},\quad f'(x)=-\frac{x+2}{(x-2)^3},\quad f''(x)=\frac{2x+8}{(x-2)^4}.$$

    (a) Identify all intercepts, horizontal and vertical asymptotes, relative extrema and points of inflection. Show all supporting work.
    (b) Sketch the graph using the information obtained in part (a) above.

15. Perform the indicated integration.

    (a) $\displaystyle\int\left(3e^x-\frac{2}{x}+\sqrt[3]{x^4}\right)dx$ (b) $\displaystyle\int\left(\frac{4}{\cos^2 x}+\cos x\right)dx$

    (c) $\displaystyle\int_1^4\frac{x^2-x+1}{\sqrt{x}}\,dx$ (d) $\displaystyle\int x^2\left(1+2x^3\right)\,dx$

16. Find the function f that has derivative $f'(x)=2x+1$ and passes through the point $(1,5)$.

17. (a) Sketch the region whose area can be computed using the definite integral $\displaystyle\int_0^4 1+\sqrt{x}\,dx$.

    (b) Evaluate the area of the region. Show supporting work.