(Marks)

(NOTE: Some minor modifications have been made to better suit the 1998 course. The altered questions are marked with an asterisk.)

(3×2) 1. Find $\frac{dy}{dx}$ for:

(a) $y = \ln(\arctan(x^3))$

(b) $y = x \arcsin(2x) + \operatorname{arcsec}(e^x)$

(5×7) 2. Evaluate the following integrals:

(a) $\int \sin^5(2x)\cos^3(2x)\,dx$

(b) $\int \frac{dx}{x(x+2)}$

(c) $\int x\arctan(x)\,dx$

(d) $\int \frac{x^3\,dx}{\sqrt{4x^2-9}}$

(e) $\int \frac{e^x\,dx}{7+e^{2x}}$

(f)* $\int \frac{dx}{\sqrt{x^2-9}}$

(g)* $\int \frac{x+1}{(x-1)(x^2-4)}\,dx$

(3×2) 3. Calculate the following limits

(a) $\lim_{x\to 1} \frac{x-e^{x-1}}{(x-1)^2}$

(b) $\lim_{x\to\infty} \left(\frac{x-1}{x+2}\right)^x$

(4×2) 4. Evaluate or show divergence:

(a) $\int_{1/2}^{\infty} \left(\frac{1}{x} - \frac{2}{2x+1}\right) dx$

(b) $\int_{-1}^{2} \frac{x+1}{(x(x+2))^{4/3}}\,dx$

(3×2) 5. (a) The region bounded by $x = y - y^2$ and $x = 0$ is rotated about the y-axis. Find the volume of the solid of revolution.

(b) Find the volume of the solid of revolution of the region bounded by $y = x^4$, $x = 1$, $y = 0$ is rotated about the y-axis.

(5) 6. (a) $\left\{\frac{2n}{2n^2-1}\right\}$ (Justify your answers.)

i. Is this sequence monotonic?

ii. Is this sequence bounded?

iii. Does it have a limit?

(3×2) (b) Test these sequences for convergence. If they converge, find $\lim_{n\to\infty} a_n$

i. $\left\{\frac{\cos n}{\ln n}\right\}$

ii. $\left\{\frac{e^n}{n}\right\}$

(4×4) (c) Determine by using an appropriate test whether these series converge.

i. $\sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}}$

ii. $\sum_{n=1}^{\infty} \left(\frac{2}{n} - \frac{1}{n^2}\right)$

iii. $\sum_{n=1}^{\infty} \frac{\sqrt{2n+3}}{5n^2-2}$

iv. $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(4×2) 7. Are the following series divergent, absolutely convergent, or conditionally convergent? State which tests are used in each case.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n(n+1)}{(n+2)(n+3)}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2}{(2n)!}$

(4) 8. Find the interval of convergence for:

$$\sum_{n=1}^{\infty} \frac{3^n(x-2)^n}{n+1}$$

(4) 9. Find the first 5 non-zero terms for a Maclaurin series expansion of $f(x) = \sqrt{3x+4}$ centered at $x = 0$.