Examinations
from previous years

CEGEP
JOHN ABBOTT
COLLEGE

Calculus I (Science)

(Mathematics 201–103/15)

Winter 1998

1. Refer to the sketch below to evaluate the following expressions. If a value does not exist, state in which way (∞, $-\infty$, or "does not exist").

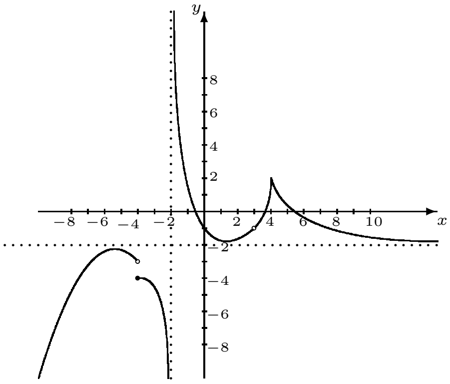
   (a) $\lim_{x\to -4} f(x)$ (b) $\lim_{x\to 3} f(x)$ (c) $\lim_{x\to 4} f(x)$

   (d) $f(-4)$ (e) $f(3)$ (f) $\lim_{x\to 2} f(x)$

   (g) $\lim_{x\to -2^-} f(x)$ (h) $\lim_{x\to \infty} f(x)$ (i) $\lim_{x\to -\infty} f(x)$

   (j) $\lim_{x\to -^-} f(x)$ (k) $\lim_{x\to -2} f(x)$

   (l) Name a value of x for which the function f is continuous but not differentiable.



2. Calculate the following limits (if they exist). Make your answer as informative as possible: if a limit does not exist, say so; if appropriate one-sided limits exist instead, state them explicitly; if any limits are infinite, state this explicitly as well.

   (a) $\displaystyle\lim_{x\to \frac{5}{4}} \frac{4x^2-25}{2x-5}$ (b) $\displaystyle\lim_{y\to \frac{\pi}{4}} y\sin^4 y$ (c) $\displaystyle\lim_{x\to -\infty} \frac{7x^3+3x+1}{x^3-2x+3}$

   (d) $\displaystyle\lim_{x\to \infty} \frac{x^{100}+x^{99}}{x^{101}-x^{100}}$ (e) $\displaystyle\lim_{x\to 4^-} \frac{2x^2+3x-2}{x^2-3x-4}$

3. Determine the values of the constants A and B so that the function f is continuous at every real number.

$$f(x) = \begin{cases} 3x & \text{if } x \leqslant 2 \\ Ax+B & \text{if } 2 < x < 5 \\ -6x & \text{if } x \geqslant 5 \end{cases}$$

4. Use the definition of derivative to find $f'(x)$ if $f(x) = \dfrac{1}{x+1}$.

5. Use the rules of differentiation to find $\dfrac{dy}{dx} = y'$. (*Do not simplify your answers.*)

   (a) $y = x^{-\frac{1}{2}}\left(x^2+2x-5\sqrt{x}+2\right)$ (b) $y = e^{\sin(-x)} - \sin\left(e^{-x}\right)$

   (c) $y = \dfrac{\tan x}{1+\sec x}$ (d) $y = \sqrt{2x}\,\ln x$ (e) $y = \left(x^2+7\right)^{3x+1}$

6. Find $y'' = \dfrac{d^2y}{dx^2}$ for $y = \left(x^3+1\right)^{\frac{1}{3}}$.

7. Find the slope of the curve $xy^3 + x^2 - 3y + 13 = 0$ at the point $(-1, 2)$.

8. Determine the point(s) at which the function f has a horizontal tangent for $f(x) = 2x^3 + 3x^2 - 12x$.

9. An astronaut standing on the moon throws a rock into the air. The height of the rock is given by $s = -\dfrac{27}{10}t^2 + 27t + 6$, where s is measured in feet and t is measured in seconds.
   (a) Find expressions for the velocity and acceleration of the rock.
   (b) Find the time when the rock is at its hightest point. What is its height at this time?

10. Determine the absolute extrema of $f(x) = 15 + 12x - x^3$ on the closed interval $[-3, 5]$.

11. Given:

$$f(x) = \frac{3x^2-4}{x^3}, \qquad f'(x) = \frac{3(4-x^2)}{x^4}, \qquad f''(x) = \frac{6(x^2-8)}{x^5}.$$

    Determine all: (a) intercepts; (b) asymptotes; (c) critical values; (d) relative extrema; (e) intervals of increase/decrease; (f) intervals of upward/downward concavity; (g) points of inflection; and then (h) sketch the graph.

12. For the following problem: (a) produce an explicit function depending on one variable only whose optimization solves the problem; (b) state what the variable represents and any restrictions on it. DO NOT SOLVE THE PROBLEM.
    A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point $(7, 5)$. What should the vertices be in order that the triangle have as small an area as possible?

13. Solve the differential equation $f'(x) = 4x - 7$ if $f(-1) = 6$.

14. Evaluate:

    (a) $\displaystyle\int \left(\frac{1}{4}e^x - \frac{5}{x} + 2\right) dx$ (b) $\displaystyle\int \csc x\,(\cot x + \csc x)\,dx$

    (c) $\displaystyle\int \frac{5x^4 - 2x^2 - 6}{x^2}\,dx$ (d) $\displaystyle\int_0^{\frac{\pi}{6}} \sin x\,dx$

15. Find the shaded area if $f(x) = x(x+2)$.