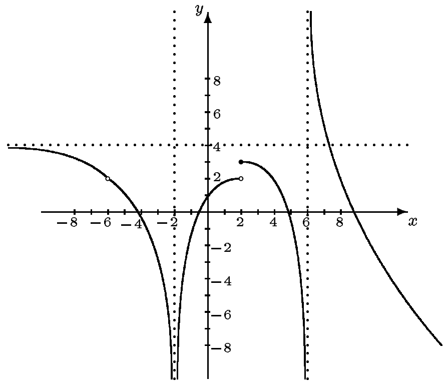
CEGEP JOHN ABBOTT COLLEGE

CALCULUS I (SCIENCE)

(MATHEMATICS 201–NYA/??)

1. Refer to the sketch below to evaluate the following expressions. If a value does not exist, state in whch way ($+\infty$, $-\infty$, or "does not exist").



(a) $\lim_{x\to-6} f(x)$ (b) $\lim_{x\to-\infty} f(x)$ (c) $\lim_{x\to+\infty} f(x)$

(d) $\lim_{x\to-2} f(x)$ (e) $\lim_{x\to2^-} f(x)$ (f) $\lim_{x\to2^+} f(x)$

(g) $\lim_{x\to2} f(x)$ (h) $\lim_{x\to0} f(x)$ (i) $\lim_{x\to5^+} f(x)$

(j) $\lim_{x\to6} f(x)$ (k) $f(-6)$ (l) $f(2)$

2. Calculate the following limits (if they exist). Make your answer as informative as possible: if a limit does not exist, say so; if appropriate one-sided limits exist instead, state them explicitly; if any limits are infinite, state this explicitly as well.

(a) $\displaystyle\lim_{x\to0}\frac{\sqrt{9-x}-3}{x}$ (b) $\displaystyle\lim_{x\to+\infty}\frac{5x^4-6x^3+3x-7}{3x^4-2x+6}$

(c) $\displaystyle\lim_{x\to-5}\frac{2x^2+7x-15}{x+5}$ (d) $\displaystyle\lim_{x\to3^-}\frac{3x^2+2x-8}{x-3}$

(e) $\displaystyle\lim_{x\to0}\frac{1-\cos x}{1-x}$

3. Let $f(x)=\begin{cases}x^2+ax & \text{if } x<1\\ x+b & \text{if } x\geqslant 1.\end{cases}$ Find values of a and b that make $f(x)$ continuous at $x=1$.

4. For each of the following statements, say whether it is true or not; justify your response.
   (a) A function can be continuous everywhere, but not differentiable at $x=1$.
   (b) A function can be differentiable everywhere, but not continuous at $x=1$.

5. For each of the following functions, calculate the derivative $\dfrac{dy}{dx}$. Do not simplify your answers.

(a) $y=3x^{13}+\dfrac{3}{5x^4}-5\sqrt[4]{x^3}+e^{\pi}$ (b) $y=2\sec x^5+3\tan^5 x$

(c) $y=\dfrac{\cos 4x}{\sqrt{9+x^2}}$ (d) $y=\left(e^{1-\cot x}+2\right)(\sin 3x-1)$

(e) $y=3^{2x-1}-\log_3(5x)+\ln^2(3x)$ (f) $y=(3-\cos x)^{x^2}$

(Hint: Use logarithmic differentiation if appropriate.)

6. Please construct the equation for the line tangent to the graph of $y=x+\sqrt{x}$ at the point where $x=9$.

7. Please determine all points where the graph of the function $y=x^3-6x^2-15x+3$ has a horizontal tangent.

8. Given $x^2(x^2+y^2)=y^2+19$, (a) find $\dfrac{dy}{dx}$; (b) find the equation for the tangent line to the curve at $(2,1)$.

9. Given $y=\dfrac{1}{x}+\tan x$, find $\dfrac{d^2y}{dx^2}$.

10. State the limit definition of the derivative. Use this definition to find the derivative $f'(x)$ for $f(x)=x^2-5$.

11. The position of a particle is given by the equation $s=t^3-6t^2+9t$ where t is measured in seconds and s is measured in meters.
    (a) Determine the average velocity from $t=2$ to $t=4$.
    (b) Find the instantaneous velocity at time t.
    (c) What is the instantaneous velocity at time $t=2$ (*i.e.*, after 2 seconds)?
    (d) When is the particle at rest (not moving)?

12. Evaluate the following integrals:

(a) $\displaystyle\int\left(7\pi-\frac{3}{x}+e^x+\frac{3}{x^2}\right)dx$ (b) $\displaystyle\int\left(\frac{x^3-8x}{2x}\right)dx$

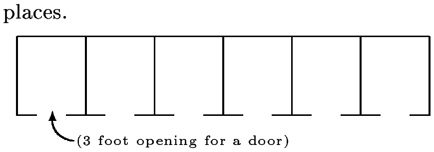
(c) $\displaystyle\int\left(\csc^3\vartheta-8\csc\vartheta\right)\sin\vartheta\,d\vartheta$ (d) $\displaystyle\int_{\frac{\pi}{2}}^{\pi}(\sin\varphi-2\cos\varphi)\,d\varphi$

13. Given that $y''=2e^x+x$, $y'(0)=3$ and $y(0)=1$, find y.

14. Find the area under the curve $y=\sqrt{x}-\dfrac{1}{x}$, above the x-axis, between $x=1$ and $x=4$.

15. Find all (relative and absolute) maximum and minimum values of the function $f(x)=x^5-5x^3$.

16. A radiologist wants to construct six identical x-ray rooms (see the sketch below). She has 200 feet of gyprock wall sheets and six doors (each 3 feet wide) available. Find the dimensions of the rooms so that the area is maximized. Give your answer correct to 2 decimal places.



17. Sketch the graph of a function $f(x)$ having all of the following properties.

(a) $\lim_{x\to2^-} f(x)=-\infty$ (b) $\lim_{x\to-2^+} f(x)=\infty$

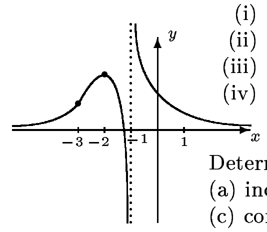
(c) $\lim_{x\to-\infty} f(x)=1$ (d) $\lim_{x\to\infty} f(x)=1$

(e) $\lim_{x\to4} f(x)$ exists but $f(x)$ is not continuous at $x=4$.

(f) $f(0)=0$ (g) $f'(0)$ does not exist

(h) $f''(x)<0$ for $x<-2$ (i) $f''(x)>0$ for $x>4$

18. Below is a sketch of the graph of a function $y=f(x)$. Notice that the graph shows the following features of the function:



(i) $y=0$ is a horizontal asymptote,
(ii) $x=-1$ is a vertical asymptote,
(iii) $(-2,3)$ is a relative maximum point of f,
(iv) $(-3,2)$ is a point of inflection of f.

Determine the intervals for which $f(x)$ is:
(a) increasing; (b) decreasing;
(c) concave up; (d) concave down.

19. Sketch the graph of $f(x)=3x^4-2x^3+1$. At what points (x,y) does the function have relative extrema and points of inflection? On what intervals is the function increasing? On what intervals is the function decreasing? Make sure your graph clearly illustrates all these features.