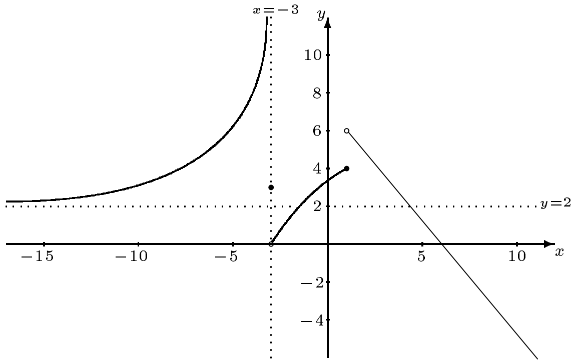
CEGEP JOHN ABBOTT COLLEGE

Calculus I (Science)

(Mathematics 201–NYA/??)

1. Determine each of the following limits if possible. If the limit does not exist explain and/or use ∞ or $-\infty$ as appropriate. (Show your work.)
   (a) $\lim_{x\to 0} \dfrac{\sin^2 x}{2x}$ (b) $\lim_{x\to -1} \dfrac{x^2+2x+1}{x^2-x-2}$
   (c) $\lim_{x\to 6} \dfrac{\sqrt{6-x}-2}{2-x}$ (d) $\lim_{x\to 3^-} \dfrac{|x-3|}{(x-3)^2}$
   (e) $\lim_{x\to\infty} \dfrac{3x^3+4x+2}{5-x-x^3}$
   (f) Use the graph of f given below to find the limits.



   $\lim_{x\to-\infty} f(x)$ $\lim_{x\to-3^+} f(x)$ $\lim_{x\to\infty} f(x)$
   $\lim_{x\to 2^-} f(x)$ $\lim_{x\to 2^+} f(x)$ $\lim_{x\to 2} f(x)$

2. *Use the definition of derivative* to find $f'(x)$ if
$$f(x) = \frac{1}{1+x}.$$

3. Use the *definition of continuity at a point* to determine if
$$f(x) = \begin{cases} x-4 & \text{if } x \leqslant 2 \\ x^2-6 & \text{if } x > 2 \end{cases}$$
is continuous at $x = 2$.

4. Find the value of k such that
$$g(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ k & \text{if } x = 3 \end{cases}$$
is continuous at $x = 3$.

5. Find the derivative of the following functions. Do not simplify your answers.
   (a) $y = 3x^2 - \dfrac{4}{\sqrt[3]{x}} + \pi$ (b) $f(x) = \dfrac{3x-5}{\sqrt{x}+1}$
   (c) $y = 5\sec^2(3x) + \tan(3x^2)$
   (d) $g(x) = e^{-x^2}\ln(1+x^2)$ (e) $y = \sqrt{x^2-a^2}$ (where a is constant)

6. Use logarithmic differentiation to find $\dfrac{dy}{dx}$, if $y = \left(x^2+1\right)^{\frac{1}{x}}$. Do not simplify your answer.

7. Let $f(x) = x^2e^{2x}$.
   (a) Determine if $f'(x)$ and simplify.
   (b) Determine any values of x where the tangent line to the graph of f is horizontal.

8. If $y = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$, find y'' and write in simplified form as a single fraction without negative exponents.

9. Given $x^2+y^2 = 3y$.
   (a) Find $\dfrac{dy}{dx}$ at the point $(-\sqrt{2}, 2)$.
   (b) Find an equation for the tangent line to the curve at $(-\sqrt{2}, 2)$.

10. Determine the absolute maximum and absolute minimum values of $f(x) = 12x - x^3$ on $[0, 4]$.

11. Given $f(x) = x^{\frac{2}{3}}(x+5)$,
$$f'(x) = \frac{5(x+2)}{3x^{\frac{1}{3}}} \quad \text{and} \quad f''(x) = \frac{10(x-1)}{9x^{\frac{4}{3}}}.$$
   (a) Find intercepts, relative extrema, and inflection points, if any.
   (b) Use the information in (a) to sketch the graph of f.

12. Given: $V = \frac{4}{3}\pi r^3$ (Volume of a balloon formula) $S = 4\pi r^2$ (Surface area of a balloon formula).
The volume of a large balloon is decreasing at a constant rate of 50 ft^3/min.
   (a) How fast is the radius r of the balloon decreasing at the instant the radius is 4 ft?
   (b) How fast is the surface area of S decreasing at the instant the radius is 4 ft?

13. Determine the coordinates of the point on the graph of $y = \dfrac{6}{x}$, $x > 0$ which is closest to the origin by minimizing the square of the distance between the origin and a typical point on the graph.

14. Find the following integrals:
   (a) $\displaystyle\int \left(2x - 1 + \frac{3}{x} - \frac{4}{x^2}\right) dx$ (b) $\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \tfrac{1}{2}\cos\vartheta\, d\vartheta$
   (c) $\displaystyle\int (e^x + x^e)\, dx$ (d) $\displaystyle\int_1^2 3x(x^2+5)\, dx$

15. Find the area of the region enclosed by $y = x^2 + \sec^2 x$ and $y = 0$ from $x = 0$ to $x = \frac{\pi}{4}$.