CEGEP JOHN ABBOTT COLLEGE

Calculus I (Science)

(Mathematics 201–NYA/??)

1. Evaluate the following limits. Assign a finite number, or $+\infty$, or $-\infty$, or state that the limit does not exist. (*No decimals*)

   (a) $\displaystyle\lim_{x\to 5}\frac{2x^2-7x-15}{x^2-25}$ (b) $\displaystyle\lim_{x\to 3^+}\frac{2x-3}{x^2-2x-3}$

   (c) $\displaystyle\lim_{x\to 2^+}\sqrt{2-x}$ (d) $\displaystyle\lim_{x\to -\infty}\frac{5x^3-2x^2+7}{x+5}$

   (e) $\displaystyle\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$ (f) $\displaystyle\lim_{x\to \frac{\pi}{4}}\frac{2\sin^2 x+9\cos(4x)}{27\tan x}$

2. (a) State a limit definition for the derivative, $f'(x)$, of a given function, $f(x)$.
   (b) Use your definition from part (a) to find $f'(x)$ for $f(x)=\dfrac{1}{2x+1}$.

3. Using the definition of continuity at a point, determine if
$$f(x)=\begin{cases}1-x & \text{if } x\leqslant 2\\ x^2-2x & \text{if } x>2\end{cases}$$
is continuous at $x=2$.

4. Find the value of c such that
$$f(x)=\begin{cases}cx+1 & \text{if } x\leqslant -3\\ cx^2-1 & \text{if } x>-3\end{cases}$$
is continuous at $x=-3$.

5. Find the derivative for each of the following functions. (*Do not simplify your answers.*)

   (a) $y=2x^6-\dfrac{4}{x^3}+6\sqrt[3]{x^4}+e^8$

   (b) $y=\dfrac{2x-1}{x^2+1}$ (c) $y=\log_2(x^3+3x^2+2)$

   (d) $s(t)=e^{\cos t}+\sin\left(t^2\right)$ (e) $g(x)=x^2\csc(3x)$

6. If $y=\ln(1+\sin x)$ find y'' and *simplify your answer.*

7. Use *logarithmic differentiation* to express $\dfrac{dy}{dx}$ in terms of x alone. *Do not simplify your answers.*

   (a) $y=\dfrac{x^5\cos x}{\sqrt[3]{x^4+16}}$ (b) $y=\left(x^3+2x\right)^{\sec x}$

8. If $f(x)=x\sqrt{98-x^2}$, then find:
   (a) $f'(x)$ and simplify;
   (b) the value(s) of x where the tangent line is *horizontal*;
   (c) the value(s) of x where the tangent line is *vertical.*

9. Find an equation of the tangent line to the graph of
$$3x^4y+y^3-11x^2=5$$
at the point $(-2,1)$.

10. Find the absolute maximum and absolute minimum values of $f(x)=x^3-4x^2+3$ on $[-1,2]$.

11. Given: $f(x)=\dfrac{3(x^2-4x+3)}{x^2}$,
$$f'(x)=\frac{6(2x-3)}{x^3}\quad\text{and}\quad f''(x)=\frac{6(9-4x)}{x^4}.$$
   (a) Find all intercepts, asymptotes, relative extrema and inflection points of $f(x)$.
   (b) Use the information in (a) to sketch a large well-labeled graph of $f(x)$. On your graph, indicate all intercepts, asymptotes, relative extrema and inflection points.

12. An airplane flies at a constant altitude of 4 miles toward a point directly over an observer on the ground. The speed of the plane is 200 miles per hour. Find the rate of change of the angle of elvation of the plane from the observer at the instant when the plane is 8 miles from the observer.

13. A closed rectangular box with a square base has a surface area of 150 square centimeters. If the sides of the base are x centimeters, find the dimensions of the box that will give the maximum volume. What is the maximum volume?

14. Find the following integrals.

   (a) $\displaystyle\int\left(3x^4-\frac{4}{x^3}+4\sqrt{x^3}-3^4\right)dx$

   (b) $\displaystyle\int_1^3 2x\left(x^2+1\right)^2dx$ (c) $\displaystyle\int\frac{\cos^3 x+1}{\cos^2 x}\,dx$

   (d) $\displaystyle\int\left(\frac{e^x}{5}+\frac{e^5}{x}-5\sin x\right)dx$ (e) $\displaystyle\int_0^{\frac{\pi}{3}}(1+\tan\vartheta\sec\vartheta)\,d\vartheta$